Mathematics

7.11 The student understands that the way a set of data is displayed influences its interpretation. The student is expected to:

(A) select and use an appropriate representation for presenting and displaying relationships among collected data including line plot, line graph, bar graph, stem and leaf plot, circle graph, and Venn diagrams, and justify the selection.

(B) make inferences and convincing arguments based on analysis of given or collected data.

7.12 The student uses measures of central tendency and range to describe a set of data. The student is expected to:

(A) describe a set of data using mean, median, mode and range.

(B) choose among mean, median, mode or range to describe a set of data and justify the choice for a particular situation.

Technology Applications

The student demonstrates knowledge and appropriate use of hardware components, software programs, and their connections. The student is expected to:

(1)(B) compare, contrast, and appropriately use the various input, processing, output, and primary/secondary storage devices.

(1)(C) demonstrate the ability to select and use software for a defined task according to quality, appropriateness, effectiveness, and efficiency.

(1)(E) use technology terminology appropriate to the task.

(1)(F) perform basic software application functions including, but not limited to, opening an application program and creating, modifying, printing, and saving documents.

The student uses data input skills appropriate to the task. The student is expected to:

(2)(A) demonstrate proficiency in the use of a variety of input devices such as mouse/track pad, keyboard, microphone, digital camera, printer, scanner, disk/disc, modem, CD-ROM, or joystick.

The student acquires electronic information in a variety of formats, with appropriate supervision. The student is expected to:

(5)(A) identify, create, and use files in various formats such as text, bitmapped/vector graphics, image, video, and audio files.

The student uses appropriate computer-based productivity tools to create and modify solutions to problems. The student is expected to:

(7)(G) integrate two or more productivity tools into a document including, but not limited to, tables, charts and graphs, graphics from paint or draw programs, and mail merge.

Materials

Advanced Preparation:

- Sign up for 2 to 3 days in the computer lab
- Have the **Central Tendencies and Technology** file ready for students to access in the computer lab.

For whole class demonstration:

- **Transparency: Fumble Bumbles**

For each student:

- **Football Statistics** activity sheet
- **How Do These Shapes Measure Up?** activity sheet
- **Data Mix-Up** performance assessment

ENGAGE

The Engage portion of the lesson is designed to create student interest in the concepts addressed. Technology is not being used in this phase since the focus of this activity is to remind students of the measures of central tendency. This part of the lesson is designed for groups of 2 students or individual investigation.

1. Display **Transparency 1: Fumble Bumbles** so that it is visible to all students.
2. Students should read the problem and solve for the mean, median, and mode. Provide math vocabulary glossaries or dictionaries for students who may need to refresh their memories on these terms.
3. Debrief the activity using the Facilitation Questions.

Facilitation Questions

- How did you determine the mean for this set of data?
  Answers may vary. Lead students in the development/review of the vocabulary word by using a graphic organizer, such as a vocabulary model, or creating a word wall for Probability and Statistics. Some students may know the algorithm for finding the mean. Other students may make a picture to find the mean, as addressed in the grade 6 mathematics TEKS. The mean is 1.375 or 1.4 fumbles.

Vocabulary Model Example

(Word)	***(Definition)***
Mean	*The sum of the numbers in a set of data divided by the number of pieces of data.*
(Examples) *Average* *3, 5, 4, 8* $3+5+4+8=20$ $20\div 4=5$	***(Non-Examples)*** *Median* *Mode*

- How did you find the median for this set of data?
  Answers may vary. Lead students in the development/review of the vocabulary word by using a graphic organizer or creating a word wall for Probability and Statistics. The median is 1.5 fumbles.
- How did you find the mode for this set of data?
  Answers may vary. Lead students in the development/review of the vocabulary word by using a graphic organizer or creating a word wall for Probability and Statistics. The mode is 2 fumbles.
- Which measure of data would the Texans prefer the media to report? Why?
  Answers may vary. The Texans would probably prefer the mean to be reported since it is the lowest of the three.

EXPLORE

The Explore portion of the lesson provides the student with an opportunity to be actively involved in the exploration of the mathematical concepts addressed. This part of the lesson is designed for groups of 2 students or individual investigation.

1. Distribute **Football Statistics** to each student.
2. Students should work to complete the worksheet using the accompanying spreadsheet document **Central Tendencies and Technology** under the tab labeled **Football Statistics**.
3. When monitoring students in the lab, use the facilitation questions.
4. Before students print, lead them to set the print area so only one page prints.

Facilitation Questions

- How do you name a cell?
  Name a cell using the letter of the column and the number of the row (A1, D32, etc.).
- How do you highlight a range of cells?
  Click the mouse and hold inside the first cell and drag to the last cell needed.
- How do you format cells?
  Use the mouse and right click or choose "Format" in the menu bar and cells from the pull down menu.
- How do you put a range of cell locations when entering a formula?
  Type the cell locations using the keyboard. Start with the first cell needed followed by a colon and the last cell needed (A1:A5).
- How did you make your prediction? Did any information help you make your prediction?
  Answers may vary. Students may discuss that the mode gave them a clue to include both of the 52 yard amounts in the first 7 games.
- What strategies did you use for choosing the numbers to put in for the 7 games?
  Answers may vary.
- Which measure of central tendency is the easiest to determine first?
  The mode is the easiest one to recognize in a set of data because one only looks at frequencies.
- If your mean is too high, how might you change your data choices?
  Answers may vary. Students may recognize that lower numbers need to be included or that higher numbers need to be replaced.

EXPLAIN

The Explain portion of the lesson is directed by the teacher to allow the students to formalize their understanding of the TEKS addressed in the lesson.

1. Debrief the **Football Statistics** activity using the facilitation questions.

Facilitation Questions

- How did the spreadsheet allow you to manipulate data?
  By using a formula to find each central tendency, any yardage can be changed in the list and each central tendency is automatically recalculated.
- How did the spreadsheet assist you in interpreting data?
  Answers may vary. Some students may say that the spreadsheet helped them to focus on the concept of mean, median, and mode since they did not have to think about the math operations involved.
- With a mean of 31, what can you conclude about the data set?
  The numbers will cluster in the middle or the numbers will vary. If the numbers vary, they must include high and low numbers to average out.
- How did the median of 24 help narrow your choices?
  Answers may vary, but lead students to put the numbers in numerical order. Discuss observations.
- If the yards from the other 3 games were included in the data set, how would you predict the mean would change? The median? The mode?
  Answers may vary. After students make predictions, point out that the 3 remaining numbers cluster within the same range, so the mean may not change much. The median should be higher since the 3 numbers would come in the middle of the existing data. The mode isn't affected.
- Were you surprised by the results? Why?
  Answers may vary. Students should explain their reasoning for being surprised.
- Were there times when the technology made the task easier? Why?
  Answers may vary. Some students may say that not having to calculate the math with paper/pencil made the task easier.
- Are there times when the technology made the task more difficult? Why?
  Answers may vary. Students may say that formatting the cells and inputting formulas made the task more difficult.

ELABORATE

The Elaborate portion of the lesson provides an opportunity for the student to apply the concepts of the TEKS within a new situation. This part of the lesson is designed for groups of 2 students or individual investigation.

1. Distribute **How Do These Shapes Measure Up?** activity sheet to each student.
2. Tell students that in the first phases of this lesson we looked at numerical data generated from football statistics, but in this phase we will be looking at numerical data generated by measuring the dimensions of figures.

3. Students should work to complete the worksheet using the accompanying spreadsheet document **Central Tendencies and Technology** under the tab labeled **How do these shapes measure up**?
4. Before students print, lead them to set the print area so only one page prints.
5. Debrief using the facilitation questions.

Facilitation Questions

- Which set of data did you predict to have the same mean, median and mode?
  Answers may vary. Some students may notice that Set C appears to be the same height, so the mean, median and mode might be the same.
- Which set of data did you predict to have the greatest mean?
  Answers may vary. Some students may realize that the taller objects will most likely create a greater mean.
- Which set of data did you predict to have the smallest mean?
  Answers may vary. Some students may realize that the shorter objects will most likely create a smaller mean.
- How does the spreadsheet assist you in analyzing data?
  Answers may vary. Some students may say that the ability to make a graph quickly helps you visually analyze similarities and differences.
- How does the spreadsheet assist you in communicating your results?
  Answers may vary. Some students may say that the spreadsheet helps them organize the data into a table and display the information graphically.
- What formula did you use to find the mean?
  =AVERAGE(first cell:last cell)
- What formula did you use to find the median?
  =MEDIAN(first cell:last cell)
- What formula did you use to find the mode?
  =MODE(first cell:last cell)
- Which set of figures has the same mean, median and mode?
  Set C
- Which set has no mode?
  Set B
- Which set has the same median and mode?
  Set A and C
- Which data set has the greatest mean?
  Set C
- Which data set has the smallest mean?
  Set B
- How can looking at the figures in Set A help you determine the central tendencies?
  Answers may vary. Visually examine the figures and use reasonableness to draw conclusions. For example, two of the figures in Set A appear to have the same height so the mode will be equal to the height of Figure 1 and 2 and so will the median since one of these heights will be the middle number. The mean will be slightly more because figure 3 will raise the average.

Facilitation Questions

- How can looking at the figures in Set B help you determine the central tendencies?
  Answers may vary. Visually look at the figures and use reasonableness to draw conclusions. For example, all of the heights in Set B are different, so that set won't have a mode. The median will be the height of Figure 5. The mean may be close to the median since the figures on either side of Figure 5 will balance out the average.
- How can looking at the figures in Set C help you determine the central tendencies?
  Answers may vary. Visually look at the figures and use reasonableness to draw conclusions. For example, all of the figures in Set C appear to be the same height, so they will have the same mean, median and mode.
- How might combining the data sets affect the mean? The median? The mode? Why?
  mean – Answers may vary. One possible answer is that the mean will be 1.25 or maybe slightly lower since the heights in the other groups are slightly higher and lower than 1.25
  median – Answers may vary. One possible answer is the median will be similar to Set C since figures 6-9 seem to have the same height and would fall in the middle of the data.
  mode – Answers may vary. The students will most likely say 1.25 since no other height occurs more than the height of the figures in Set C.
- How did the mean, median and mode of the lengths/diameters compare to that of the heights?
  Answers may vary. None of the sets has a mode. Set B has the highest mean. The median for Set C was the same for its length and its height.
- How do the bar graphs help you to interpret the data?
  Answers may vary. Students should recognize that the graph provides a visual representation, but caution them about misleading statistics.
- Why do you think we are using a bar graph instead of a circle graph?
  Answers may vary. Circle graphs are typically used with data represented as percentages.

EVALUATE

The Evaluate portion of the lesson provides the student with an opportunity to demonstrate his or her understanding of the TEKS addressed in the lesson.

1. Distribute **Data Mix-Up** activity sheet to each student.
2. Upon completion of **Data Mix-Up** activity sheet, the teacher should use a rubric to assess student understanding of the concepts addressed in this lesson.

Answers and Error Analysis for selected response questions:

Question Number	TEKS	Correct Answer	Conceptual Error	Conceptual Error	Procedural Error	Procedural Error	Guess
1	7.12B	B	A	C			D
2	7.12A	B	C	D	A		
3	7.12B	C	A	B			D
4	7.11B	C	A	B			D

Football Statistics *(Possible Answers)*

In 2004 Cory Bradford was a receiver for the Texans. He received the ball in 12 out of the 16 games played by the team. The total yards received during each of the first 10 games are shown below.

24 9 52 32 5 52 27 13 65 38

If Cory Bradford's mean, median and mode for receptions during the first 7 games were 31, 24, and 52 (when rounded to the nearest whole number), which of the above yardages represents his stats?

1. Use the spreadsheet document to help you find the yards received by Cory Bradford during the first 7 games. Follow the instructions on the spreadsheet given in each of the colored boxes.

2. If the yards from the other 3 games were included in the data set, how would you predict
   a. the mean would change?
      Answers may vary. Since the numbers remaining cluster together, students may suggest the mean will stay the same.

   b. the median would change?
      Answers may vary. Help the students realize they will average the 2 numbers in the middle.

   c. the mode would change?
      The mode won't change since 52 is the only repeating number.

3. Use the spreadsheet to calculate the mean, median, and mode for all 10 games. Set up a table beside or below the existing information.

4. How close were your predictions to the actual mean, median and mode? Explain similarities and differences.
   Answers may vary. Students should be detailed in explanations.

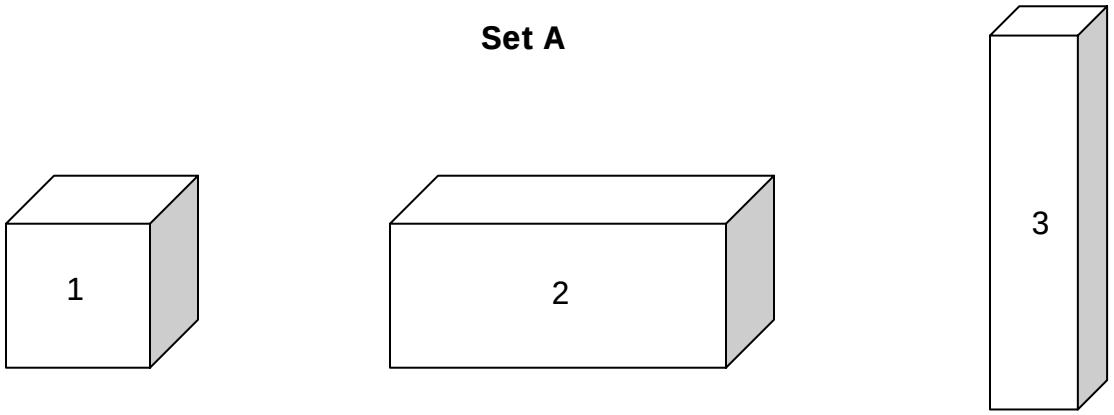
5. Print the file when finished. Be sure to ask your teacher for any special directions before printing.

How Do These Shapes Measure Up? *(Possible Answers)*

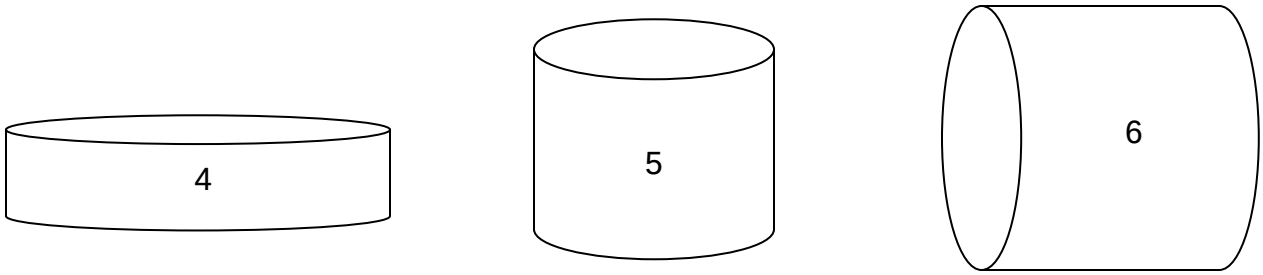
1. Look at each set of figures below. Make a prediction about the mean, median, and mode for the heights of each set. For which set of data do you predict the mean, median, and mode to be the same? Which set do you predict to have the greatest mean? Which set do you predict to have the smallest mean?

   Answers may vary. Students should use the size of the figures to make predictions.

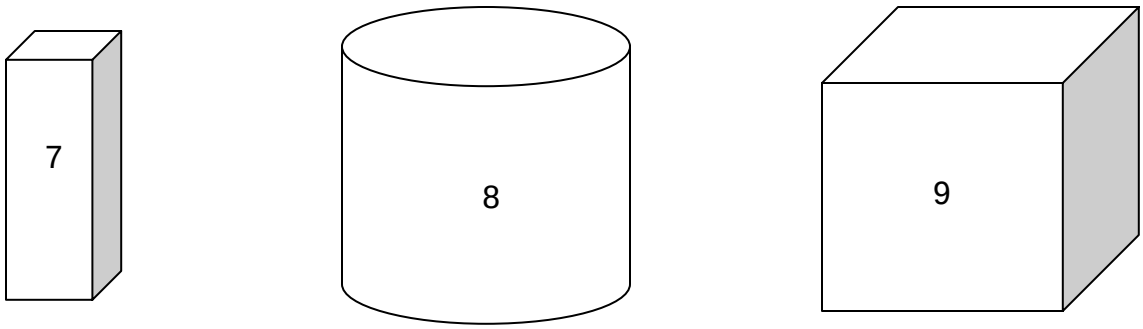
Set A



Set B



Set C



2. Measure the height of each figure. Round measurements to the nearest $\frac{1}{4}$ inch.

   (For example, for any measurement between 1 and 1.25, round to 1.25.)

3. Use the spreadsheet document to
   a. organize data
   b. determine the mean, median and mode using formulas for the heights of each set
   c. chart the mean, median and mode for the heights of each set

4. Use this information to answer the following questions.
   a. Which set of figures has the same mean, median and mode?
      Set C
   b. Which set has no mode?
      Set B
   c. Which set has the same median and mode?
      Set A and C
   d. Which data set has the greatest mean?
      Set C
   e. Which data set has the smallest mean?
      Set B
   f. How can looking at the figures help you determine the central tendencies?
      Answers may vary. Lead students to realize they could visually look at the figures and use reasonableness to draw conclusions. For example, all of the figures in Set C appear to be the same height, so they will have the same mean, median and mode. All of the heights in Set B are different, so that set won't have a mode.
   g. How would combining the data sets affect the mean? The median? The mode?
      mean – *Answers may vary. One possible answer is that the mean will be 1.25 or maybe slightly lower since the heights in the other groups are slightly higher and lower than 1.25*
      median – *Answers may vary.*
      mode – *Answers may vary. The students will most likely say 1.25 since no other height occurs more than the height of the figures in Set C.*

5. How different do you think the data sets would be if you measured the lengths or diameters of the figures? What would be similar? What would be different? Explain your reasoning.
   Answers may vary. Students should reason about the differences in the data sets by looking at the sides.

6. Create a new table to the side of the current spreadsheet in order to find the mean, median, and mode of the lengths or diameters for each set of figures. Be sure to round measurements to the nearest $\frac{1}{4}$ inch.

7. Print the file when finished. Be sure to ask your teacher for any special directions before printing.

Data Mix-Up *(Possible Answers)*

Mr. Tucker gave his students the following data from the 2004 football season.

The Houston Texans played 16 games in 2004. The numbers in the table represent the total passing yards by David Carr, the quarterback, for each game.

229	215
313	164
233	201
228	157
372	167
266	220
276	139
245	114

Each student had to create a data set of passing yards for the losing games and a data set of passing yards for the winning games using the clues provided.

Clue 1: The Texans had 2 fewer wins in 2004 than losses.
Clue 2: The mean passing yards for the losing data set is less than the mean passing yards for the winning data set.
Clue 3: All of the passing yard totals for the winning games are in the same hundreds group except for 1.
Clue 4: The range for the passing yards of the losing games is 258 and of the winning games is in the one hundred range.
Clue 5: The smallest value in both data sets is in the one hundred range.

The data sets for 2 students are shown below.

Marissa	
Losses	Wins
313	372
276	266
245	233
229	228
215	220
167	201
164	114
157	
139	

Sheldon	
Losses	Wins
372	276
313	266
245	233
229	228
215	220
167	201
164	139
157	
114	

Use the clues and a spreadsheet to make your own data set. Find the mean, median and mode using formulas for each of your data sets. Compare your results to the given student results to decide which student is correct. Justify your reasoning.

Sheldon is correct. See spreadsheet answer key for work.

Fumble Bumbles

A fumble in a football game is the failure to hold or handle the ball properly. If the opposing team recovers the fumble, they gain possession of the ball at the precise location of the recovery. Fumbles are many times key turning points in a game and could cause the team a loss.

In the 2004 football season, the Houston Texans played 16 games. The chart below shows the number of fumbles made by the Texans.

Game Date	# of Fumbles
Sept. 12	2
Sept. 19	4
Sept. 26	2
Oct. 3	1
Oct. 10	0
Oct. 17	2
Oct. 31	2
Nov. 7	0
Nov. 14	3
Nov. 21	0
Nov. 28	0
Dec. 5	2
Dec. 12	1
Dec. 19	0
Dec. 26	2
Jan. 2	1

Which measure of data (mean, median, or mode) would the Texans prefer the media report? Explain your reasoning.

Football Statistics

In 2004 Cory Bradford was a receiver for the Texans. He received the ball in 12 out of the 16 games played by the team. The total yards received during each of the first 10 games are shown below.

24 9 52 32 5 52 27 13 65 38

If Cory Bradford's mean, median and mode for receptions during the first 7 games were 31, 24, and 52 (when rounded to the nearest whole number), which of the above yardages represents his stats?

1. Use the spreadsheet document to help you find the yards received by Cory Bradford during the first 7 games. Follow the instructions on the spreadsheet given in each of the colored boxes.

2. If the yards from the other 3 games were included in the data set, how would you predict
   a. the mean would change?

   b. the median would change?

   c. the mode would change?

3. Use the spreadsheet to calculate the mean, median, and mode for all 10 games. Set up a table beside or below the existing information.

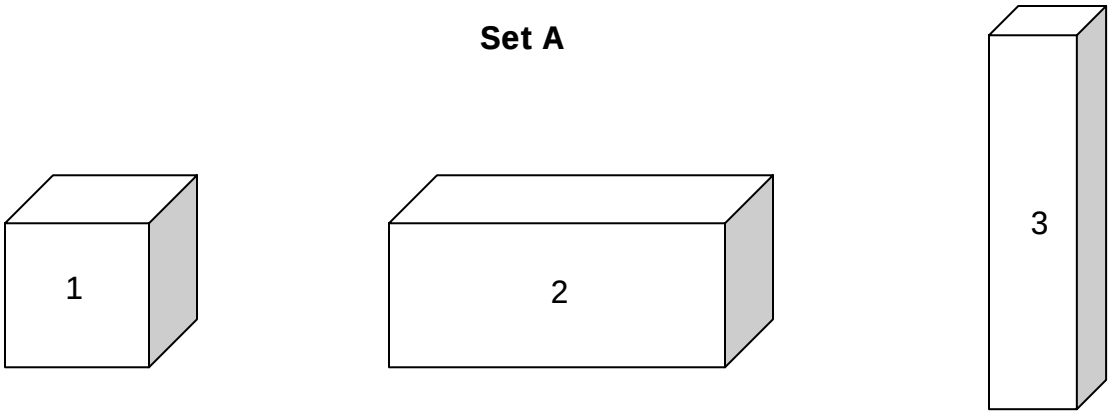
4. How close were your predictions to the actual mean, median and mode? Explain similarities and differences.

5. Print the file when finished. Be sure to ask your teacher for any special directions before printing.

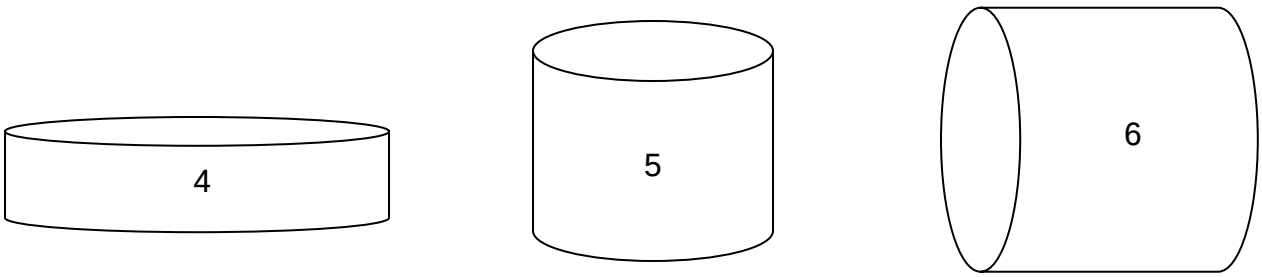
How Do These Shapes Measure Up?

1. Look at each set of figures below. Make a prediction about the mean, median, and mode for the heights of each set. For which set of data do you predict the mean, median and mode to be the same? Which set do you predict to have the greatest mean? Which set do you predict to have the smallest mean?

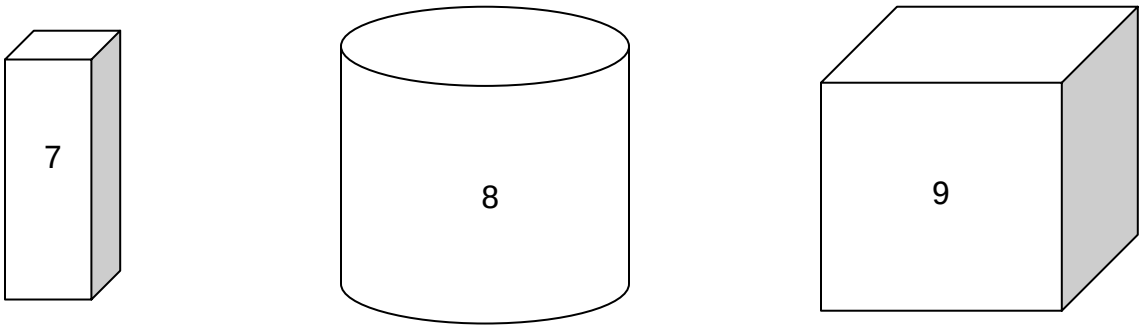
Set A



Set B



Set C



2. Measure the height of each figure. Round measurements to the nearest $\frac{1}{4}$ inch.

3. Use the spreadsheet document to
   a. organize data.
   b. find the mean, median and mode using formulas for the heights of each set.
   c. chart the mean, median and mode for the heights of each set.

4. Use the information to answer the following questions.

   d. Which set of figures has the same mean, median and mode?

   e. Which set has no mode?

   f. Which set has the same median and mode?

   g. Which data set has the greatest mean?

   h. Which data set has the smallest mean?

   i. How can looking at the figures help you determine the central tendencies?

   j. How would combining the data sets affect the mean? The median? The mode?

      mean –

      median –

      mode –

5. How different do you think the data sets would be if you measured the lengths or diameters of the figures? What would be similar? What would be different? Explain your reasoning.

6. Create a new table to the side of the current spreadsheet in order to find the mean, median, and mode of the lengths or diameters for each set of figures. Be sure to round measurements to the nearest $\frac{1}{4}$ inch. Chart the data.

7. Print the file when finished. Be sure to ask your teacher for any special directions before printing.

Data Mix-Up

Mr. Tucker gave his students the following data from the 2004 football season.

The Houston Texans played 16 games in 2004. The numbers in the table represent the total passing yards by David Carr, the quarterback, for each game.

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Losses	Wins
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313	266
245	233
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215	220
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157	
114	

Use the clues and a spreadsheet to make your own data set. Find the mean, median and mode using formulas for each of your data sets. Compare your results to the given student results to decide which student is correct. Justify your reasoning.

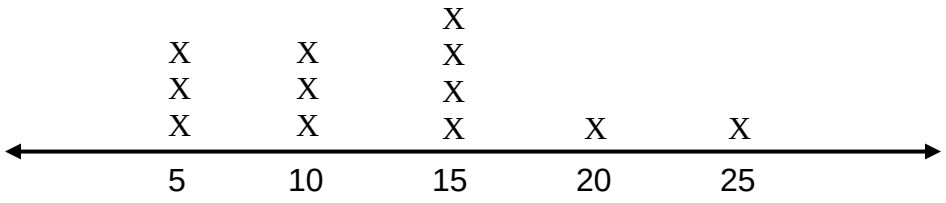
1. The table shows the number of points Menu scored during the first 5 basketball games.

Game	Points Scored
1	15
2	11
3	18
4	12
5	29

If Menu wants to predict how many points he will score during the next game, which measure of the data should he use?

A Mean
B Median
C Mode
D Range

2. Mai charges $5 per hour for babysitting. She decided to chart the amount she earned on different evenings spent babysitting during the past month.



What was the median amount she earned during the month?

A $10
B $12.50
C $14
D $15

3. In his first three hours of waiting tables, Kimiko received the following tip amounts.

$2 $1.50 $2 $3.25 $5 $2.25 $12

If Kimiko wants to ask for a raise by showing his tips are not very good, which measure of central tendency should he show his boss?

A Mean
B Median
C Mode
D Range

4. To participate in an activity at the Fall Festival or purchase food items, tickets must be purchased. Below is a table that describes some booths and food items at the Fall Festival and the number of tickets needed for that booth.

Activity or Food Item	Number of Tickets
Cake Walk	3
Fishing	2
Moon Walk	4
Pony Ride	6
Ring Toss	2
Rock Climbing	7
Chips	3
Drinks	3
Hot Dogs	5
Nachos	5

If a petting zoo is added to the list above, how many tickets should the Festival organizers assigned to the petting zoo for the mean to stay the same?

A 3
B 3.5
C 4
D 5

Mathematics

7.11 The student understands that the way a set of data is displayed influences its interpretation. The student is expected to:

(A) select and use an appropriate representation for presenting and displaying relationships among collected data including line plot, line graph, bar graph, stem and leaf plot, circle graph, and Venn diagrams, and justify the selection.

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7.12 The student uses measures of central tendency and range to describe a set of data. The student is expected to:

(A) describe a set of data using mean, median, mode and range.

(B) choose among mean, median, mode or range to describe a set of data and justify the choice for a particular situation.

Materials

For whole class demonstration:

- **Transparency: Fumble Bumbles**

For each student:

- TI-73 calculator
- **Football Statistics** activity sheet
- **How Do These Shapes Measure Up?** activity sheet
- **Data Mix-Up** performance assessment

ENGAGE

The Engage portion of the lesson is designed to create student interest in the concepts addressed. Technology is not used in this phase since the focus of this activity is to remind students of the measures of central tendency. This part of the lesson is designed for groups of 2 students or individual investigation.

1. Display **Transparency 1: Fumble Bumbles** so that it is visible to all students.
2. Students should read the problem and solve for the mean, median, and mode. Provide math vocabulary glossaries or dictionaries for students who may need to refresh their memories on these terms.
3. Debrief the activity using the Facilitation Questions.

Facilitation Questions

- How did you determine the mean for this set of data?
  Answers may vary. Lead students in the development/review of the vocabulary word by using a graphic organizer, such as a vocabulary model, or creating a word wall for Probability and Statistics. Some students may know the algorithm for finding the mean. Other students may make a picture to find the mean, as addressed in the 6th grade TEKS. The mean is 1.375 or 1.4 fumbles.

Vocabulary Model Example

(Word)	***(Definition)***
Mean	*The sum of the numbers in a set of data divided by the number of pieces of data.*
(Examples) *Average of 3, 5, 4, 8* $3+5+4+8=20$ $20\div 4=5$	***(Non-Examples)*** *Median* *Mode*

- How did you find the median for this set of data?
  Answers may vary. Lead students in the development/review of the vocabulary word by using a graphic organizer or creating a word wall for Probability and Statistics. The median is 1.5 fumbles.
- How did you find the mode for this set of data?
  Answers may vary. Lead students in the development/review of the vocabulary word by using a graphic organizer or creating a word wall for Probability and Statistics. The mode is 2 fumbles.
- Which measure of data would the Texans prefer the media to report?
  Answers may vary. The Texans would probably prefer the mean to be reported since it is the lowest of the three.

EXPLORE

The Explore portion of the lesson provides the student with an opportunity to be actively involved in the exploration of the mathematical concepts addressed. This part of the lesson is designed for groups of 2 students or individual investigation.

1. Distribute **Football Statistics** activity sheet to each student and a TI-73 calculator.
2. When monitoring students thinking, use the facilitation questions.

Facilitation Questions

- What do you know about the problem?
  Possible answer: For the first 10 games, the mean is 31, the median is 24 and the mode is 52.
- What do you need to know to find a solution for the problem?
  Possible answer: I need to find the data set for the 7 games.
- Where is the information located in the calculator that you need?
  The information is in List 1 under the LIST feature of the calculator.
- What should you do if the mean is higher than the targeted mean?
  The values in the data set need to be decreased if the mean is too high.
- What should you do if the mean is lower than the targeted mean?
  The values in the data set need to be increased if the mean is too low.
- How did you make your prediction? Did any information help you make your prediction?
  Answers may vary. The mode gave them a clue to include both of the 52 yard amounts in the first 7 games.
- What strategies did you use for choosing the numbers to put in for the 7 games?
  Answers may vary.
- Which measure of central tendency is the easiest to determine first?
  The mode is the easiest one to recognize in a set of data because one only looks at frequencies.

EXPLAIN

The Explain portion of the lesson is directed by the teacher to allow the students to formalize their understanding of the TEKS addressed in the lesson.

1. Debrief the **Football Statistics** activity using the facilitation questions.

Facilitation Questions

- How did the calculator allow you to manipulate data?
  By using the LIST feature and allowing the calculator to find each measure of central tendency, any yardage can be changed in the list and each measure of central tendency recalculated easily.
- How did the calculator assist you in interpreting data?
  Answers may vary. Some students may say that the calculator helped them to focus on the concept of mean, median, and mode since they did not have to think about the math operations involved.
- With a mean of 31, what can you conclude about the data set?
  The numbers will cluster in the middle or the numbers will vary. If the numbers vary, they must include high and low numbers to average out.
- How did the median of 24 help narrow your choices?
  Answers may vary, but lead students to put the numbers in numerical order. Discuss observations.
- If the yards from the other 3 games were included in the data set, how would you predict the mean would change? The median? The mode?
  Answers may vary. After students make predictions, point out that the 3 remaining numbers cluster within the same range, so the mean may not change much. The median should be higher since the 3 numbers would come in the middle of the existing data. The mode isn't affected.
- Were you surprised by the results? Why?
  Answers may vary. Students should explain their reasoning for being surprised.
- Were there times when the technology made the task easier? Why?
  Answers may vary. Some students may say that not having to calculate the math with paper/pencil made the task easier.
- Are there times when the technology made the task more difficult? Why?
  Answers may vary. Students may say that manipulating back and forth between the lists and calculations made the task more difficult.

ELABORATE

The Elaborate portion of the lesson provides an opportunity for the student to apply the concepts of the TEKS within a new situation. This part of the lesson is designed for groups of 2 students or individual investigation.

1. Distribute **How Do These Shapes Measure Up?** activity sheet and TI-73 calculator to each student.
2. Tell students that in the first phases of this lesson we looked at numerical data generated from football statistics, but in this phase we will be looking at numerical data generated by measuring the dimensions of figures.

3. Students should work to complete the worksheet.
4. Debrief using the facilitation questions.

Facilitation Questions

- Which set of data did you predict to have the same mean, median and mode?
  Answers may vary. Some students may notice that Set C appears to be the same height, so the mean, median and mode might be the same.
- Which set of data did you predict to have the greatest mean?
  Answers may vary. Some students may realize that the taller objects will most likely create a greater mean.
- Which set of data did you predict to have the smallest mean?
  Answers may vary. Some students may realize that the shorter objects will most likely create a smaller mean.
- How does the calculator assist you in analyzing data?
  Answers may vary. Some students may say that the ability to make a graph quickly helps you visually analyze similarities and differences.
- How does the calculator assist you in communicating your results?
  Answers may vary. Some students may say that the calculator helps them organize the data into a table and display the information graphically.
- What formula did you use to find the mean?
  MEAN(L_1)
- What formula did you use to find the median?
  MEDIAN(L_1)
- What formula did you use to find the mode?
  MODE(L_1)
- Which set of figures has the same mean, median and mode?
  Set C
- Which set has no mode?
  Set B
- Which set has the same median and mode?
  Set A and C
- Which data set has the greatest mean?
  Set C
- Which data set has the smallest mean?
  Set B
- How can looking at the figures in Set A help you determine the central tendencies?
  Answers may vary. Visually examine the figures and use reasonableness to draw conclusions. For example, two of the figures in Set A appear to have the same height so the mode will be equal to the height of Figure 1 and 2 and so will the median since one of these heights will be the middle number. The mean will be slightly more because figure 3 will raise the average.

Facilitation Questions

- How can looking at the figures in Set B help you determine the central tendencies?
  Answers may vary. Visually examine the figures and use reasonableness to draw conclusions. For example, all of the heights in Set B are different, so that set won't have a mode. The median will be the height of Figure 5. The mean may be close to the median since the figures on either side of Figure 5 will balance out the average.
- How can looking at the figures in Set C help you determine the central tendencies?
  Answers may vary. Visually examine the figures and use reasonableness to draw conclusions. For example, all of the figures in Set C appear to be the same height, so they will have the same mean, median and mode.
- How would combining the data sets affect the mean? The median? The mode?
  mean – Answers may vary. One possible answer is that the mean will be 1.25 or maybe slightly lower since the heights in the other groups are slightly higher and lower than 1.25
  median – Answers may vary. One possible answer is the median will be similar to Set C since figures 6-9 seem to have the same height and would fall in the middle of the data.
  mode – Answers may vary. The students will most likely say 1.25 since no other height occurs more than the height of the figures in Set C.
- How different did you think the data sets would be for the lengths/diameters?
  Answers may vary. Students should reason about the differences in the data sets by looking at the sides.
- How did the mean, median and mode of the lengths/diameters compare to that of the heights?
  Answers may vary. None of the sets has a mode. Set B has the highest mean. The median for Set A and Set C are the same for both.
- How do the bar graphs help you to interpret the data?
  Answers may vary. Students should recognize that the graph provides a visual representation, but caution them about misleading statistics.
- Why do you think we are using a bar graph instead of a circle graph?
  Answers may vary. Circle graphs are typically used with data represented as percentages.

EVALUATE

The Evaluate portion of the lesson provides the student with an opportunity to demonstrate his or her understanding of the TEKS addressed in the lesson.

1. Distribute **Data Mix-Up** activity sheet to each student.
2. Upon completion of **Data Mix-Up** activity sheet, the teacher should use a rubric to assess student understanding of the concepts addressed in this lesson.

Answers and Error Analysis for selected response questions:

Question Number	TEKS	Correct Answer	Conceptual Error	Conceptual Error	Procedural Error	Procedural Error	Guess
1	7.12B	B	A	C			D
2	7.12A	B	C	D	A		
3	7.12B	C	A	B			D
4	7.11B	C	A	B			D

Football Statistics *(Possible Answers)*

In 2004 Cory Bradford was a receiver for the Texans. He received the ball in 12 out of the 16 games played by the team. The total yards received during each of the first 10 games is shown below, but the yards are not listed in a particular order.

24 9 52 32 5 52 27 13 65 38

If Cory Bradford's mean, median and mode for receptions during the first 7 games were 31, 24, and 52 (when rounded to the nearest whole number), which of the above yardages represent his stats?

1. Make a prediction for the yards received in the first 7 games. Justify your reasoning.

   Answers will vary. Encourage students to use reasoning when making predictions.

2. Use the TI-73 calculator and the given information to help you find the yards received by Cory Bradford during the first 7 games. Follow the instructions below.

   a. Input the data using the **LIST** feature.

      Press [LIST].
      Input the 7 yards one by one into L_1.
      Press [2nd][MODE] to return to the home screen.

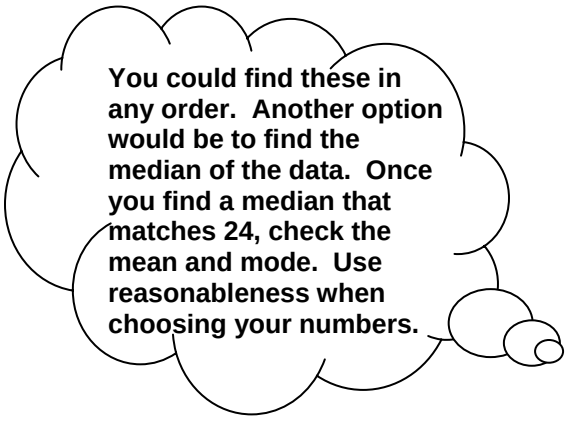
   b. Find the mean of the data using the **STAT** feature. Record your trials in the table on the next page.

      Press [2nd][LIST] to access the STAT menu.
      Press [▶] to arrow over to MATH.
      Press [▼] to arrow down to mean(
      Press [ENTER].
      Press [2nd][LIST] L_1 [ENTER].
      Press [ENTER].

Think strategically when choosing the 7 yards. If the 7 yards chosen doesn't yield 31, go back to the list and modify it. Find the mean again for the new list.

	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Trial 7	Trial 8	Trial 9
1									
2									
3									
4									
5									
6									
7									
mean									

c. Once you get 31 for the mean of a data set, check the median and mode.



To check the median:

Press 2nd LIST to access the STAT menu.
Press ▶ to arrow over to MATH.
Press ▼ to arrow down to median(
Press ENTER.
Press 2nd LIST L_1 ENTER.
Press ENTER.

To check the mode:

Press 2nd LIST to access the STAT menu.
Press ▶ to arrow over to MATH.
Press ▼ to arrow down to mode(
Press ENTER.
Press 2nd LIST L_1 ENTER.
Press ENTER.

d. Record the yards for the first 7 games below.

5, 9, 13, 24, 52, 52, 65

e. How many trials did it take before finding the yards for the 7 games?
Answers will vary. If more columns are needed for recording trials, have students draw more tables on a sheet of paper.

f. What strategies did you use to help you choose the numbers for each trial?
Answers will vary. Hopefully, answers will include reasonableness.

3. If the yards from the other 3 games were included in the data set, how would you predict
   a. the mean would change?
   Answers will vary. Since the numbers left cluster together, students may suggest the mean will stay the same.

   b. the median would change?
   Answers will vary. Help the students realize they will average the 2 numbers in the middle.

   c. the mode would change?
   The mode won't change since 52 is the only repeating number.

4. Use the TI-73 to calculate the mean, median, and mode for all 10 games. Record below.

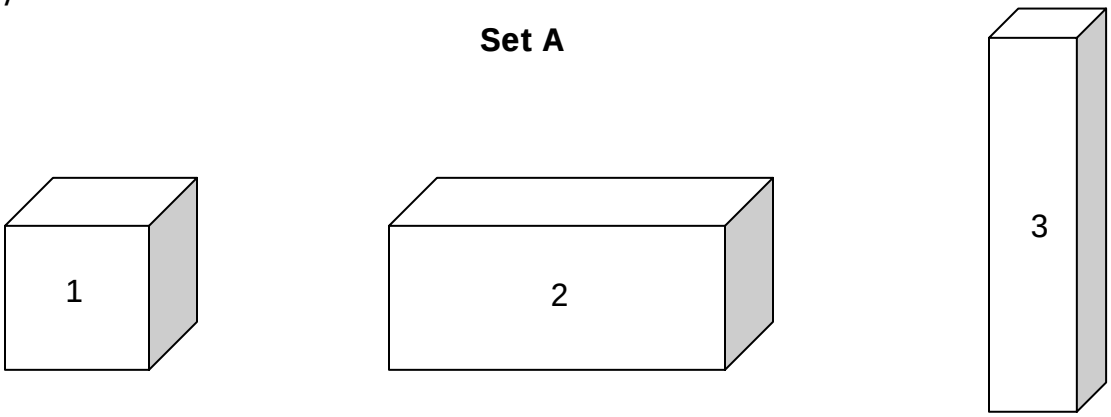
   Mean __*31.7*__ Median __*29.5*__ Mode __*52*__

5. How close were your predictions to the actual mean, median and mode? Explain similarities and differences.
   Answers will vary. Students should be detailed in explanations.

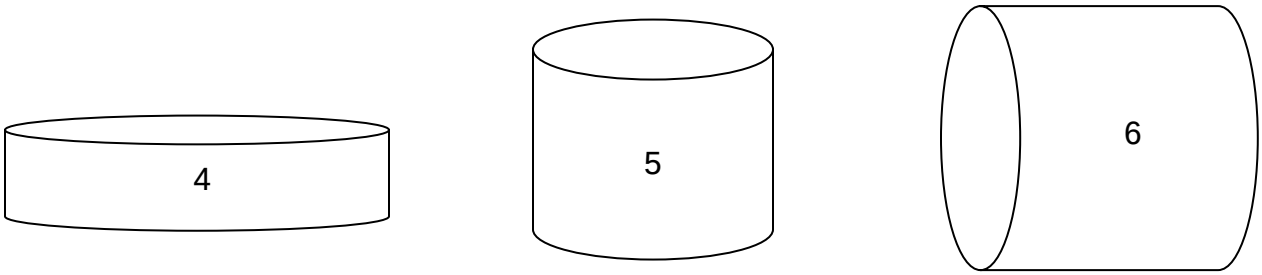
How do these shapes measure up? *(Possible Answers)*

1. Look at each set of figures below. Make a prediction about the mean, median, and mode for the heights of each set. For which set of data do you predict the mean, median and mode to be the same? Which set do you predict to have the greatest mean? Which set do you predict to have the smallest mean?
   Answers will vary. Students should use the size of the figures to make predictions.

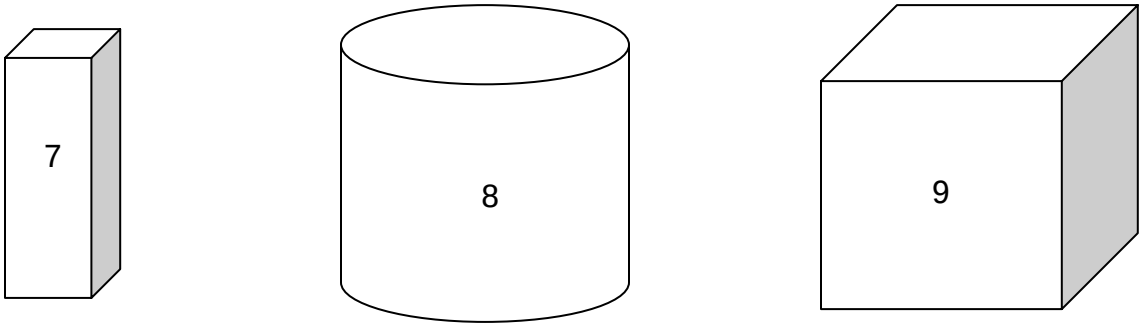
Set A



Set B



Set C



2. Measure the height of each figure. Round measurements to the nearest $\frac{1}{4}$ inch.
   Record in the chart under #4.

3. Input the height data for each set of figures using the **LIST** feature.
   Set A – L_1 Set B – L_2 Set C – L_3

4. Find the mean, median, and mode for each set of heights. Record data in the chart.

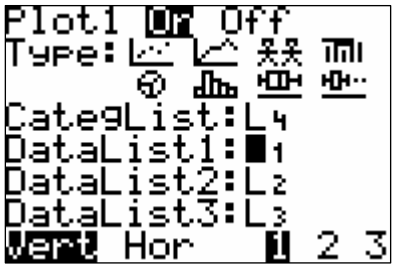
Set A	Height
1	*0.75*
2	*0.75*
3	*2*
Mean	*1.17*
Median	*0.75*
Mode	*0.75*

Set B	Height
4	*0.5*
5	*1*
6	*1.25*
Mean	*0.92*
Median	*1*
Mode	*None*

Set C	Height
7	*1.25*
8	*1.25*
9	*1.25*
Mean	*1.25*
Median	*1.25*
Mode	*1.25*

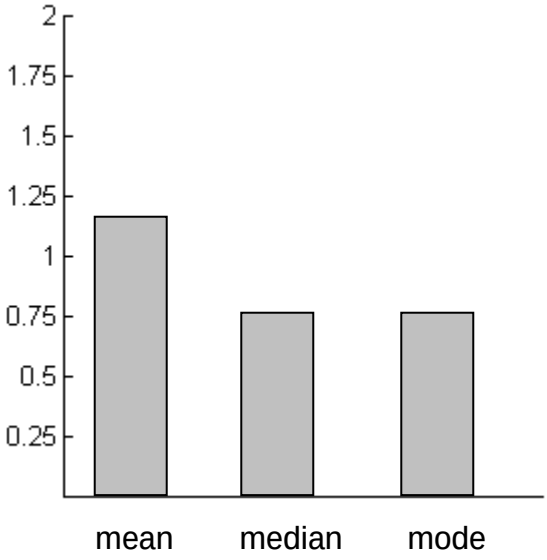
5. Input the mean, median and mode for each set of data using the **LIST** feature.
   Set A – L_4 Set B – L_5 Set C – L_6

6. Create a bar graph for the mean, median and mode of each set of heights.
   Sketch what you see.
   For each set:
   Press [2nd][Y=][ENTER].
   With the cursor blinking on ON, press [ENTER].
   Press [▼] to arrow down to the next row. Press [▶] to arrow over to the bar graph). Press [ENTER].
   Since the measures of central tendency for Set A were in L_4, choose L_4 for the CategList. To do this, press [▼] to arrow down to the CategList row. Press [2nd][LIST] and select L_4. Press [ENTER].
   Your screen should look like this:

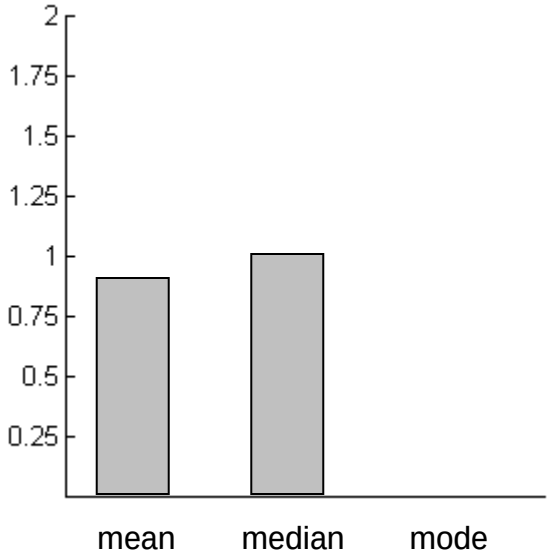


   Press [ZOOM] and arrow down to ZoomStat to see the graph.
   Sketch your graph on the next page. Repeat the process for Sets B and C.

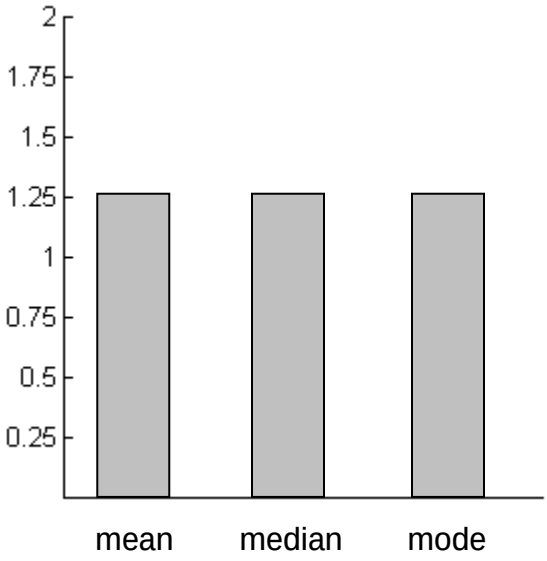
Set A



Set B



Set C



7. Use the information to answer the following questions.
    a. Which set of figures has the same mean, median and mode?
    Set C
    b. Which set has no mode?
    Set B
    c. Which set has the same median and mode?
    Sets A and C
    d. Which data set has the greatest mean?
    Set C

e. Which data set has the smallest mean?
Set B

f. How can looking at the figures help you determine the central tendencies?
Answers will vary. Lead students to realize they could visually look at the figures and use reasonableness to draw conclusions. For example, all of the figures in Set C appear to be the same height, so they will have the same mean, median and mode. All of the heights in Set B are different, so that set won't have a mode.

g. How would combining the data sets affect the mean? The median? The mode?
mean – *Answers will vary. One possible answer is that the mean will be 1.25 or maybe slightly lower since the heights in the other groups are slightly higher and lower than 1.25*

median – *Answers will vary.*

mode – *Answers will vary. The students will most likely say 1.25 since no other height occurs more than the height of the figures in Set C.*

8. How different do you think the data sets would be if you measured the lengths or diameters of the figures? What would be similar? What would be different? Explain your reasoning.
Answers will vary. Students should reason about the differences in the data sets by looking at the sides.

9. Measure the lengths or diameters for each set of figures. Be sure to round measurements to the nearest $\frac{1}{4}$ inch. Record in the chart under #10.

10. Input the length/diameter data for each set of figures using the **LIST** feature.
Set A – L_1 Set B – L_2 Set C – L_3
Find the mean, median, and mode. Record data in the chart.

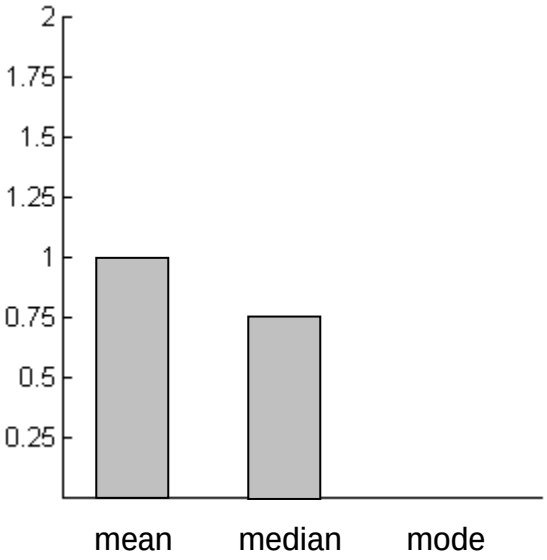
Set A	Length / Diameter
1	*0.75*
2	*1.75*
3	*0.5*
Mean	*1*
Median	*0.75*
Mode	*None*

Set B	Length / Diameter
4	*2*
5	*1.25*
6	*1.5*
Mean	*1.58*
Median	*1.5*
Mode	*None*

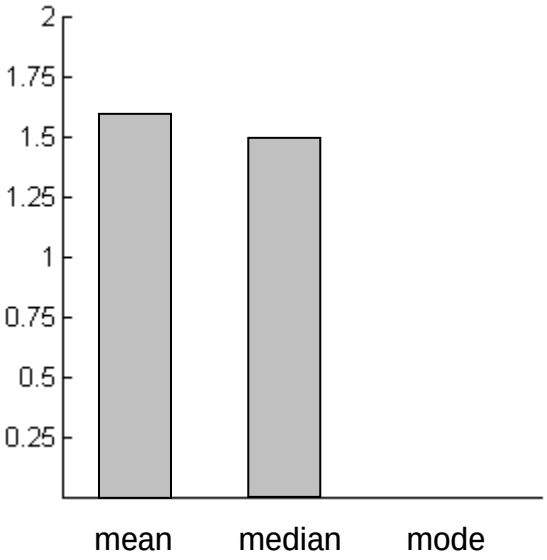
Set C	Length / Diameter
7	*0.5*
8	*1.5*
9	*1.25*
Mean	*1.08*
Median	*1.25*
Mode	*None*

11. Create a bar graph for each set of lengths/diameters. Sketch what you see.

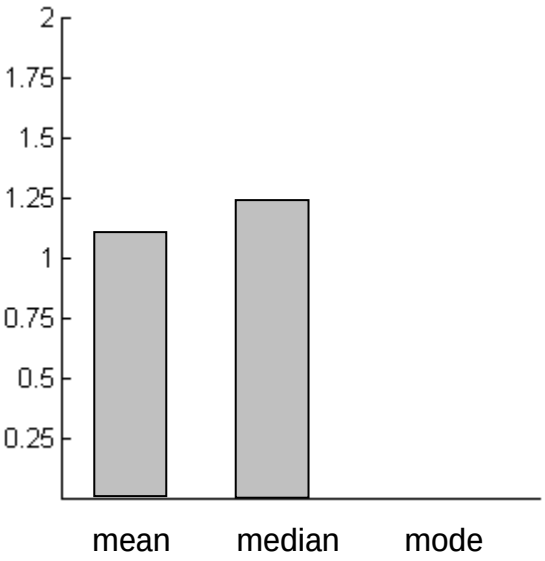
Set A



Set B



Set C



Data Mix-Up *(Possible Answers)*

Mr. Tucker gave his students the following data from the 2004 football season.

The Houston Texans played 16 games in 2004. The numbers in the table represent the total passing yards by David Carr, the quarterback, for each game.

229	215
313	164
233	201
228	157
372	167
266	220
276	139
245	114

Each student had to create a data set of passing yards for the losing games and a data set of passing yards for the winning games using the clues provided.

Clue 1: The Texans had 2 fewer wins in 2004 than losses.
Clue 2: The mean passing yards for the losing data set is less than the mean passing yards for the winning data set.
Clue 3: All of the passing yard totals for the winning games are in the same hundreds group except for 1.
Clue 4: The range for the passing yards of the losing games is 258 and of the winning games is in the one hundred range.
Clue 5: The smallest value in both data sets is in the one hundred range.

The data sets for 2 students are shown below.

Marissa	
Losses	Wins
313	372
276	266
245	233
229	228
215	220
167	201
164	114
157	
139	

Sheldon	
Losses	Wins
372	276
313	266
245	233
229	228
215	220
167	201
164	139
157	
114	

Use the clues and a TI-73 calculator to make your own data set. Find the mean, median and mode for each of your data sets. Compare your results to the given student results to decide which student is correct. Justify your reasoning.

Sheldon is correct.

Fumble Bumbles

A fumble in a football game is the failure to hold or handle the ball properly. If the opposing team recovers the fumble, they gain possession of the ball at the precise location of the recovery. Fumbles are many times key turning points in a game and could cause the team a loss.

In the 2004 football season, the Houston Texans played 16 games. The chart below shows the number of fumbles made by the Texans.

Game Date	# of Fumbles
Sept. 12	2
Sept. 19	4
Sept. 26	2
Oct. 3	1
Oct. 10	0
Oct. 17	2
Oct. 31	2
Nov. 7	0
Nov. 14	3
Nov. 21	0
Nov. 28	0
Dec. 5	2
Dec. 12	1
Dec. 19	0
Dec. 26	2
Jan. 2	1

Which measure of data (mean, median, or mode) would the Texans prefer the media report? Explain your reasoning.

Football Statistics

In 2004 Cory Bradford was a receiver for the Texans. He received the ball in 12 out of the 16 games played by the team. The total yards received during each of the first 10 games is shown below, but the yards are not listed in a particular order.

24 9 52 32 5 52 27 13 65 38

If Cory Bradford's mean, median and mode for receptions during the first 7 games were 31, 24, and 52 (when rounded to the nearest whole number), which of the above yardages represents his stats?

1. Make a prediction for the yards received in the first 7 games. Justify your reasoning.

2. Use the TI-73 calculator and the given information to help you find the yards received by Cory Bradford during the first 7 games. Follow the instructions below.
   a. Input the data using the **LIST** feature.
      Press [LIST].
      Input the 7 yards one by one into L_1.
      Press [2nd][MODE] to return to the home screen.

   b. Find the mean of the data using the **STAT** feature. Record your trials in the table on the next page.
      Press [2nd][LIST] to access the STAT menu.
      Press [▶] to arrow over to MATH.
      Press [▼] to arrow down to mean(
      Press [ENTER].
      Press [2nd][LIST] L_1 [ENTER].
      Press [ENTER].

Think strategically when choosing the 7 yards. If the 7 yards chosen doesn't yield 31, go back to the list and modify it. Find the mean again for the new list.

	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Trial 7	Trial 8	Trial 9
1									
2									
3									
4									
5									
6									
7									
mean									

c. Once you get 31 for the mean of a data set, check the median and mode.

You could find these in any order. Another option would be to find the median of the data. Once you find a median that matches 24, check the mean and mode. Use reasonableness when choosing your numbers.

To check the median:

Press 2nd LIST to access the STAT menu.
Press ▶ to arrow over to MATH.
Press ▼ to arrow down to median(
Press ENTER.
Press 2nd LIST L_1 ENTER.
Press ENTER.

To check the mode:

Press 2nd LIST to access the STAT menu.
Press ▶ to arrow over to MATH.
Press ▼ to arrow down to mode(
Press ENTER.
Press 2nd LIST L_1 ENTER.
Press ENTER.

d. Record the yards for the first 7 games below.

e. How many trials did it take before finding the yards for the 7 games?

f. What strategies did you use to help you choose the numbers for each trial?

3. If the yards from the other 3 games were included in the data set, how would you predict
   a. the mean would change?

   b. the median would change?

   c. the mode would change?

4. Use the TI-73 to calculate the mean, median, and mode for all 10 games. Record below.

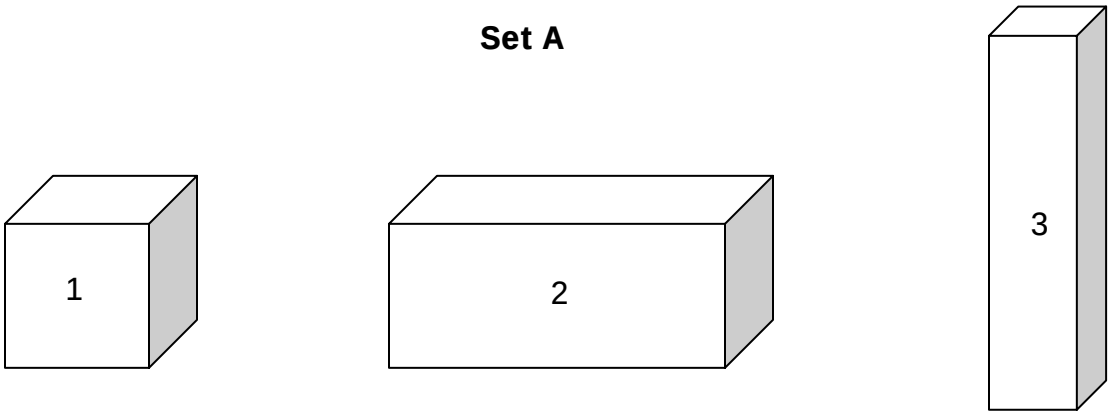
   Mean _____ Median _____ Mode _____

5. How close were your predictions to the actual mean, median and mode? Explain similarities and differences.

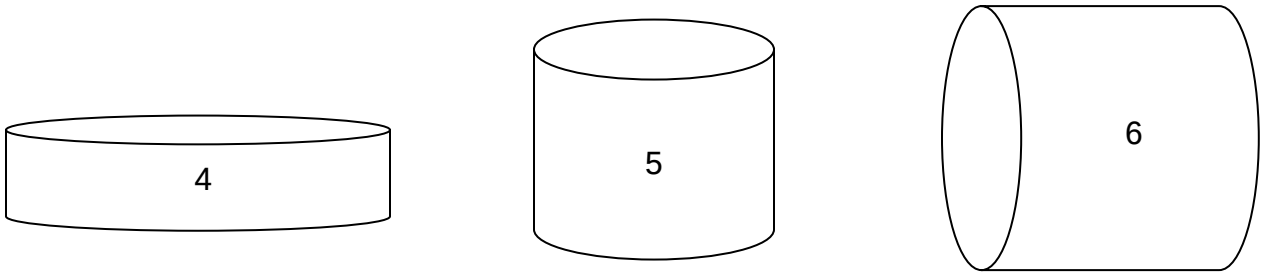
How do these shapes measure up?

1. Look at each set of figures below. Make a prediction about the mean, median, and mode for the heights of each set. For which set of data do you predict the mean, median and mode to be the same? Which set do you predict to have the greatest mean? Which set do you predict to have the smallest mean?

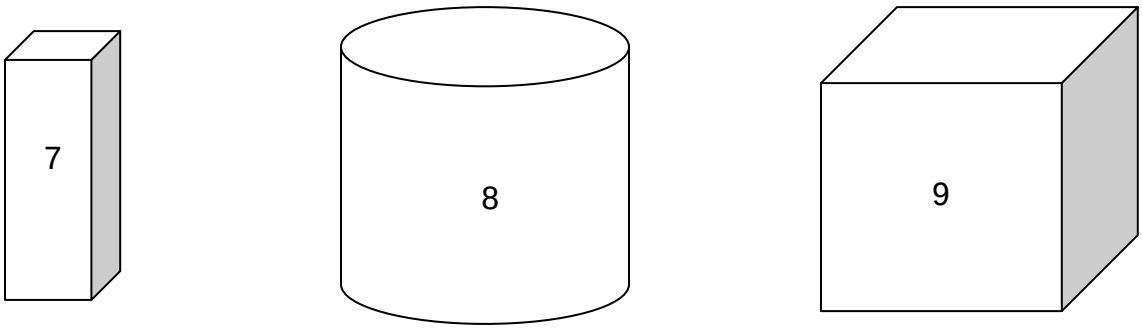
Set A



Set B



Set C



2. Measure the height of each figure. Round measurements to the nearest $\frac{1}{4}$ inch.

   Record in the chart under #4.

3. Input the height data for each set of figures using the **LIST** feature.
   Set A – L_1 Set B – L_2 Set C – L_3

4. Find the mean, median, and mode for each set of heights. Record data in the chart.

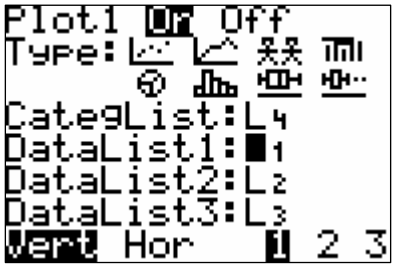
Set A	Height
1	
2	
3	
Mean	
Median	
Mode	

Set B	Height
4	
5	
6	
Mean	
Median	
Mode	

Set C	Height
7	
8	
9	
Mean	
Median	
Mode	

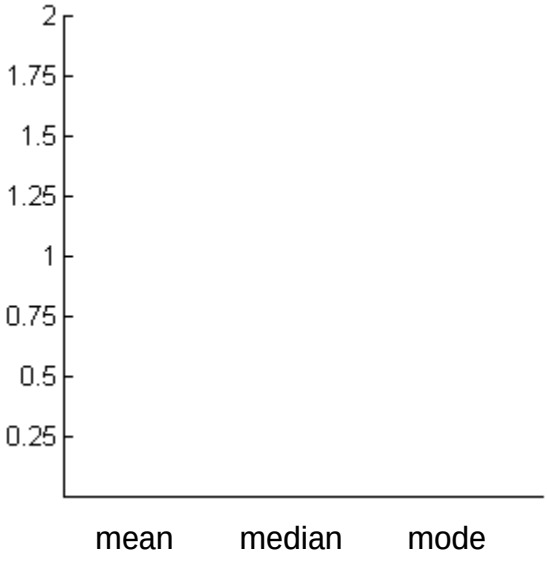
5. Input the mean, median and mode for each set of data using the **LIST** feature.
   Set A – L_4 Set B – L_5 Set C – L_6

6. Create a bar graph for the mean, median and mode of each set of heights. Sketch what you see.
   For each set:
   Press [2nd][Y=][ENTER].
   With the cursor blinking on ON, press [ENTER].
   Press [▼] to arrow down to the next row. Press [▶] to arrow over to (the bar graph). Press [ENTER].
   Since the measures of central tendency for Set A were in L_4, choose L_4 for the CategList. To do this, press [▼] to arrow down to the CategList row. Press [2nd][LIST] and select L_4. Press [ENTER].
   Your screen should look like this:

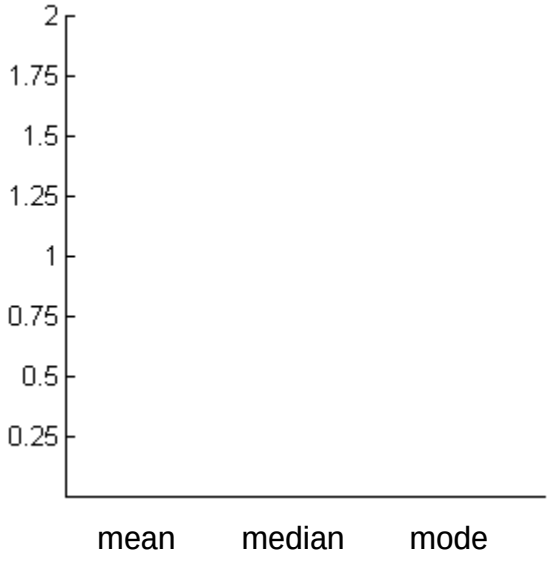


   Press [ZOOM] and arrow down to ZoomStat to see the graph.
   Sketch your graph on the next page. Repeat the process for Sets B and C.

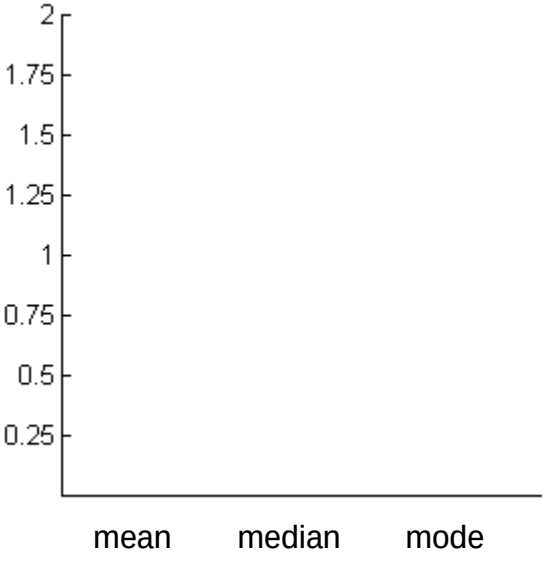
Set A



Set B



Set C



7. Use the information to answer the following questions.

    a. Which set of figures has the same mean, median and mode?

    b. Which set has no mode?

    c. Which set has the same median and mode?

d. Which data set has the greatest mean?

e. Which data set has the smallest mean?

f. How can looking at the figures help you determine the central tendencies?

g. How would combining the data sets affect the mean? The median? The mode?

mean –

median –

mode –

8. How different do you think the data sets would be if you measured the lengths or diameters of the figures? What would be similar? What would be different? Explain your reasoning.

9. Measure the lengths or diameters for each set of figures. Be sure to round measurements to the nearest $\frac{1}{4}$ inch. Record in the chart under #10.

10. Input the length/diameter data for each set of figures using the **LIST** feature.
Set A – L_1 Set B – L_2 Set C – L_3
Find the mean, median, and mode. Record data in the chart.

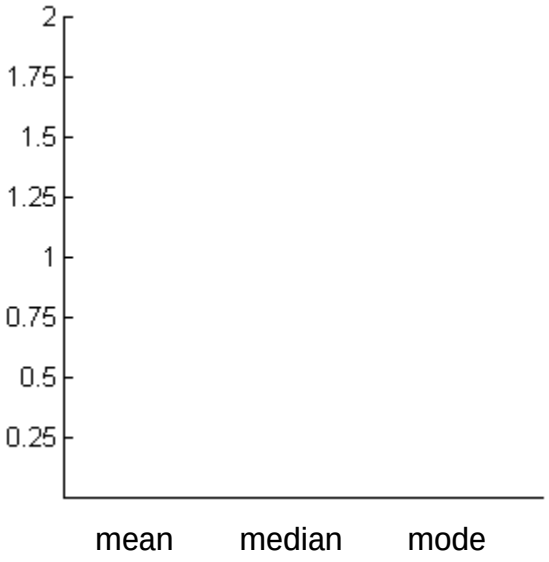
Set A	Length/ Diameter
1	
2	
3	
Mean	
Median	
Mode	

Set B	Length/ Diameter
4	
5	
6	
Mean	
Median	
Mode	

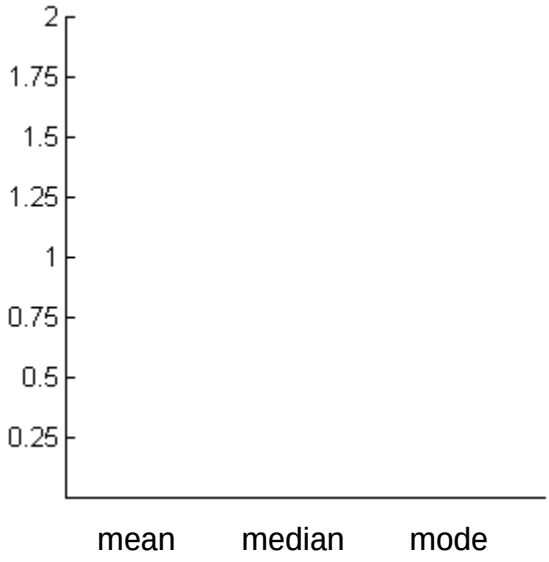
Set C	Length/ Diameter
7	
8	
9	
Mean	
Median	
Mode	

11. Create a bar graph for each set of lengths/diameters. Sketch what you see.

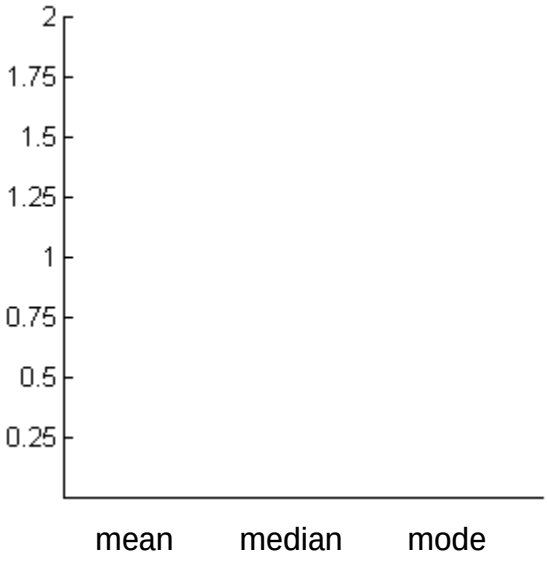
Set A



Set B



Set C



Data Mix-Up

Mr. Tucker gave his students the following data from the 2004 football season.

The Houston Texans played 16 games in 2004. The numbers in the table represent the total passing yards by David Carr, the quarterback, for each game.

229	215
313	164
233	201
228	157
372	167
266	220
276	139
245	114

Each student had to create a data set of passing yards for the losing games and a data set of passing yards for the winning games using the clues provided.

Clue 1: The Texans had 2 fewer wins in 2004 than losses.
Clue 2: The mean passing yards for the losing data set is less than the mean passing yards for the winning data set.
Clue 3: All of the passing yard totals for the winning games are in the same hundreds group except for 1.
Clue 4: The range for the passing yards of the losing games is 258 and of the winning games is in the one hundred range.
Clue 5: The smallest value in both data sets is in the one hundred range.

The data sets for 2 students are shown below.

Marissa	
Losses	Wins
313	372
276	266
245	233
229	228
215	220
167	201
164	114
157	
139	

Sheldon	
Losses	Wins
372	276
313	266
245	233
229	228
215	220
167	201
164	139
157	
114	

Use the clues and a TI-73 calculator to make your own data set. Find the mean, median and mode for each of your data sets. Compare your results to the given student results to decide which student is correct. Justify your reasoning.

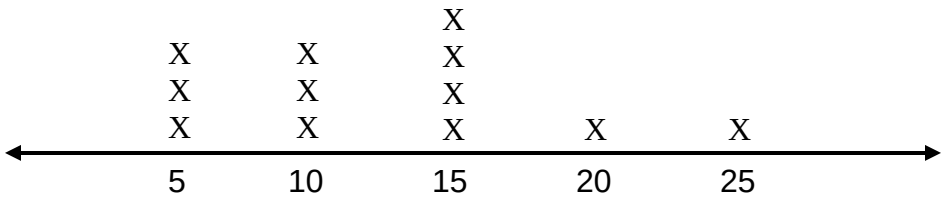
1. The table shows the number of points Menu scored during the first 5 basketball games.

Game	Points Scored
1	15
2	11
3	18
4	12
5	29

If Menu wants to predict how many points he will score during the next game, which measure of the data should he use?

A Mean
B Median
C Mode
D Range

2. Mai charges $5 per hour for babysitting. She decided to chart the amount she earned on different evenings spent babysitting during the past month.



What was the median amount she earned during the month?

A $10
B $12.50
C $14
D $15

3. In his first three hours of waiting tables, Kimiko received the following tip amounts.

$2 $1.50 $2 $3.25 $5 $2.25 $12

If Kimiko wants to ask for a raise by showing his tips are not very good, which measure of central tendency should he show his boss?

A Mean
B Median
C Mode
D Range

4. To participate in an activity at the Fall Festival or purchase food items, tickets must be purchased. Below is a table that describes some booths and food items at the Fall Festival and the number of tickets needed for that booth.

Activity or Food Item	Number of Tickets
Cake Walk	3
Fishing	2
Moon Walk	4
Pony Ride	6
Ring Toss	2
Rock Climbing	7
Chips	3
Drinks	3
Hot Dogs	5
Nachos	5

If the Fall Festival adds a petting zoo to the list above, how many tickets should the petting zoo cost for the mean to stay the same?

A 3
B 3.5
C 4
D 5

Mathematics

7.10 The student recognizes that a physical or mathematical model can be used to describe the experimental and theoretical probability of real –life. The student is expected to

(B) find the probability of independent events.

7.11 The student understands that the way a set of data is displayed influences its interpretation.

(A) select and use an appropriate representation for presenting and displaying relationships among collected data, including line plot, line graph, bar graph, stem and leaf plot, circle graph, Venn diagrams, and justify the selection.

(B) make inferences and convincing arguments based on analysis of given or collected data.

Technology Applications

The student demonstrates knowledge and appropriate use of hardware components, software programs, and their connections. The student is expected to

(1)(B) compare, contrast, and appropriately use the various input, processing, output, and primary/secondary storage devices.

(1)(C) demonstrate the ability to select and use software for a defined task according to quality, appropriateness, effectiveness, and efficiency.

(1)(E) use technology terminology appropriate to the task.

(1)(F) perform basic software application functions including, but not limited to, opening an application program and creating, modifying, printing, and saving documents.

The student uses data input skills appropriate to the task. The student is expected to

(2)(A) demonstrate proficiency in the use of a variety of input devices such as mouse/track pad, keyboard, microphone, digital camera, printer, scanner, disk/disc, modem, CD-ROM, or joystick.

The student acquires electronic information in a variety of formats, with appropriate supervision. The student is expected to

(5)(A) identify, create, and use files in various formats such as text, bitmapped/vector graphics, image, video, and audio files.

The student uses appropriate computer-based productivity tools to create and modify solutions to problems. The student is expected to

(7)(G) integrate two or more productivity tools into a document including, but not limited to, tables, charts and graphs, graphics from paint or draw programs, and mail merge.

Materials

Advanced Preparation:

- Sign up for the computer lab.
- Have the **The Teacher Helper** file ready for students to access in the computer lab.
- Make the **Teacher Helper** transparency.

For each student:

- **The Helper Dilemma** activity sheet
- **The Choir Helper** activity sheet
- **Simulation** activity sheet
- Sticky notes

For whole class demonstration:

Transparency of **Teacher Helper**

For each student group of students:

- Coin
- 10-sided number decahedron
- Assortment of spinners, polyhedral-dice, marbles, or cards for **The Choir Helper** activity or materials for students to create their own

ENGAGE

The Engage portion of the lesson is designed to create student interest in the concepts addressed. Technology is not used in this phase since the focus of this activity is to remind students of sample spaces and probability. This part of the lesson is designed for whole group instruction and groups of 2 students.

1. Display the **Teacher Helper** transparency on the overhead.
2. Read the problem as a class and ask students to take a moment to think about the problem on their own. Have students record their solutions on a sticky note.
3. Prompt the students to work with a partner to share and/or compile their thoughts and answer the questions.
4. Debrief using the facilitation questions.
5. Extend the discussion to find other probabilities such as the probability of getting an even number, the number 11, etc.

Facilitation Questions – Engage Phase

- How many students are in Mrs. Alexander's class?
  20
- How do you know?
  Answers may vary. This will hopefully lead into a discussion on sample spaces.
- What is a sample space?
  A sample space is the set of all possible outcomes for a given scenario.
- What is the sample space for this scenario?

Heads, 1	*Heads, 6*	*Tails, 1*	*Tails, 6*
Heads, 2	*Heads, 7*	*Tails, 2*	*Tails, 7*
Heads, 3	*Heads, 8*	*Tails, 3*	*Tails, 8*
Heads, 4	*Heads, 9*	*Tails, 4*	*Tails, 9*
Heads, 5	*Heads, 10*	*Tails, 5*	*Tails, 10*

- Are all of the possibilities equally likely? Why?
  Yes, there is only one head and one tail. Also, each number occurs only one time.
- What is the probability of the student assigned to Head, 6 being the helper?
  Have students refer back to the sample space. There is a $\frac{1}{20}$ *chance for the student with a Head, 6 to be the helper. Connect the sample space to finding the independent probability.*

EXPLORE

The Explore portion of the lesson provides the student with an opportunity to be actively involved in the exploration of the mathematical concepts addressed. This part of the lesson is designed for groups of 2 students.

1. Distribute a 10-sided number decahedron and a coin to each pair of students.
2. Distribute **The Helper Dilemma** activity sheet to each student.
3. The students should perform the experiment and record results.
4. Take students into the computer lab to complete the activity.
5. Use the facilitation questions when students need help.

Facilitation Questions – Explore Phase

- How do you highlight a range of cells?
  Click the mouse and hold inside the first cell and drag to the last cell needed. Let go of the mouse.
- How do you format cells?
  Use the mouse and right click or choose "Format" in the menu bar and cells from the pull down menu.
- How do you start any formula in a spreadsheet document?
  All formulas start with an equal sign.
- When inputting formulas is it better to use numerical values or cell locations? Why?
  Cell locations are better since the value will automatically change in the formula cell if any numerical values are changed in the linked cells. However, numerical values are also appropriate at times as seen in the spreadsheet.
- What do you know about the problem?
  Answers may vary. Have students verbalize the parts of the problem they know.
- What do you need to know?
  Answers may vary. Have students verbalize the parts of the problem they need to know through questioning.
- Have you worked problems like this before? Explain.
  Answers may vary. Relate the problem to prior learning and prior experiences.

EXPLAIN

The Explain portion of the lesson is directed by the teacher to allow the students to formalize their understanding of the TEKS addressed in the lesson.

1. Debrief **The Helper Dilemma**.
2. Discuss notation for probability of compound events (i.e. P(head, one)).

Facilitation Questions – Explain Phase

- What is the difference between theoretical and experimental probability?
  Answers may vary. Take this opportunity to review these topics.
- How were the experimental and theoretical probabilities the same numerically?
  Answers may vary. Depending on the experiment, some may say that the experimental probabilities were close to being equally distributed.

Facilitation Questions – Explain Phase

- How were the experimental and theoretical probabilities the same numerically?
  Answers may vary. Depending on the experiment, some may say that the experimental probabilities were equally distributed like the theoretical probabilities.
- How were the experimental and theoretical probabilities different?
  Answers may vary. Depending on the experiment, some of the combinations may have occurred more than others. Possibly discuss at this time how Mrs. Alexander should keep track of who is helper so that when repeats occur she knows to flip the coin and roll the number decahedron again.
- If the fractions were changed to percents, what would you expect the percents to total and why?
  Answers may vary. Lead students to the understanding that the experiment is a whole event, so that the percents would add to 100% and the fractions to 1 whole.
- How could we use the spreadsheet to change the fractions to percents?
  To change fractions to percents, highlight the desired cells and choose format cells.
- How could we use the spreadsheet to total the percent values?
  To find a total, insert a formula by typing "=SUM" and highlighting or typing in the desired cells.
- If Mrs. Alexander has more students enrolled in her class, how can she change or modify her procedure for finding a helper?
  Answers may vary. Students may suggest that she use a numbered cube with 12 sides. Some students may suggest that she flip a coin, roll the numbered cube and use a spinner with 3 or 4 sections. Some students may suggest that she use a deck of cards and assign each student a card from the deck.
- What can you conclude about the class where Mrs. Alexander assigned tails to girls and heads and prime numbers to boys?
  Answers may vary. Not all of the combinations in the sample space will be used for this class. This class has more girls than boys since more combinations are assigned to girls than boys.
- How could Mrs. Alexander change or modify her procedure for finding a helper in this class to eliminate the extra combinations?
  Answers may vary. Mrs. Alexander could use the coin and a bag of marbles with 4 different colors for the boys or a spinner with 4 equal sections.

Facilitation Questions – Explain Phase

- In a previous question you were asked, "If Mrs. Alexander has more students enrolled in her class, how can she change or modify her procedure for finding a helper?" If 3 items were used to find the helper, for example, a coin, a number cube and a spinner, how would the results be affected?
  Answers may vary. Discuss that a third item would increase outcomes.
- How could we use the spreadsheet to help us record and calculate the results?
  Answers may vary. To calculate theoretical probability, three columns will be needed on the spreadsheet. In the 4th column a formula will be inserted to multiply the probability of 3 events.
- Do the items always have to yield equally likely results?
  Answers may vary. Discuss with students that they do not. Have students give examples where the outcomes are not equally likely.
- How does technology assist us in communicating our results?
  Answers may vary. Some students may suggest that the graph is easier to interpret and helps by eliminating the fractions. Others may suggest that the ease in changing from fractions to percents helps in communicating results.

ELABORATE

The Elaborate portion of the lesson provides an opportunity for the student to apply the concepts of the TEKS within a new situation. This part of the lesson is designed for individual investigation.

1. Distribute **The Choir Helper** to each student.
2. Prompt students to complete #1-2 before going to the lab.
3. Prompt students to open the file **The Teacher Helper** spreadsheet and click on the tab titled **The Choir Helper**.
4. Students should complete the remainder of the worksheet using the spreadsheet.
5. Students should print the document when finished.
6. Ask Facilitation Questions as needed.

Facilitation Questions – Elaborate Phase

- How do you write a formula in a spreadsheet document?
  Formulas start with the = sign.
- How do spreadsheets help you?
  Answers may vary. Spreadsheets will do calculations needed with a formula.
- How is this activity similar and different to **The Helper Dilemma**?
  Answers may vary. Still has two simulation items, but larger sample space.

Facilitation Questions – Elaborate Phase

- How do you make predictions from results?
  Include a discussion here of scale factors. The simulation was for 50 trials, so to predict results for 100 trials use a scale factor of 2 (multiply by 2).
- How could you use a graph to show that the results for 50 trials, 100 trials, 250 trials, etc. are proportional?
  Answers may vary. Discuss with students that a graph could be made charting each individual outcome. For example, chart the results for P(tail, letter A) for 50 trials by letting the x-axis represent trials and the y-axis represent outcomes. Other outcomes for the same probability could be graphed, and a discussion of the data points should follow. The points should appear to be in a straight line that would travel through the origin. Thus, the data is proportional.

EVALUATE

The Evaluate portion of the lesson provides the student with an opportunity to demonstrate his or her understanding of the TEKS addressed in the lesson.

1. Distribute **Simulation** activity sheet to each student.
2. Upon completion of **Simulation** activity sheet, the teacher should use a rubric to assess student understanding of the concepts addresses in this lesson.

Answers and Error Analysis for selected response questions:

Question Number	TEKS	Correct Answer	Conceptual Error	Conceptual Error	**Procedural** Error	Procedural Error	**Guess**
1	7.10B	D	A		C		B
2	7.11B	B	A	D			C
3	7.11A	A	B	C			D
4	7.10B	A	D		B	C	

The Helper Dilemma – (Possible Answers)

1. Use a coin and a 10-sided number decahedron to simulate the experiment 40 times. Record your results in the frequency table.

Combination	Tally	Frequency
Head, 1		
Head, 2		
Head, 3		
Head, 4		
Head, 5		
Head, 6		
Head, 7		
Head, 8		
Head, 9		
Head, 10		

Combination	Tally	Frequency
Tail, 1		
Tail, 2		
Tail, 3		
Tail, 4		
Tail, 5		
Tail, 6		
Tail, 7		
Tail, 8		
Tail, 9		
Tail, 10		

2. Transfer your information into **The Teacher Helper** document. Follow the instructions in the orange boxes numbered 1-6.

3. Create a graph to represent the Theoretical Probability in Column B.
   - Highlight the Combinations (i.e. Head, 1) in Column A along with the data in the green cells in Column B.
   - Go to Insert Chart.
     - Choose "doughnut" for the chart type on the left-hand side.
     - Click on next twice and type in the title "Theoretical Probability."
     - Click on the tab that reads "Legend." Click in the box next to "Show Legend" so that the check mark disappears.
     - Click on the tab that reads "Data Labels." Click inside the boxes next to "Category Name and Value" so that a check mark appears in both boxes.
     - Click on finish.
     - Click and hold inside the chart. Drag the chart below the first set of data.
     - Enlarge the chart by clicking on a corner and dragging to the desired size.

(continue: The Helper Dilemma)

4. Create a graph to represent the Experimental Probability in Column I (include the Combinations such as Head, 1). Follow the same instructions as #3 except highlight the information in Columns H and I and use the title "Experimental Probability." Drag the chart next to the Theoretical Probability Chart, the first chart.

5. Print the document. Be sure to preview the pages to be printed. You may need to adjust margins so that you only print 1 or 2 pages.

6. How were the experimental and theoretical probabilities the same? Explain.
   Answers may vary. Depending on the experiment, some may say that the experimental probabilities were close to being equally distributed.

7. How were the experimental and theoretical probabilities different? Explain.
   Answers may vary. Depending on the experiment, some of the combinations may have occurred more than others. Possibly discuss at this time how Mrs. Alexander should keep track of who is the helper so that when repeats occur, she knows to flip the coin and roll the number decahedron again.

8. If the fractions were changed to percents, what would you expect the percents to total and why?
   Answers may vary. Lead students to the understanding that the experiment is a whole event, so that the percents would add to 100% and the fractions to 1 whole.

9. If Mrs. Alexander has more students enrolled in her class, how can she change or modify her procedure for finding a helper?
   Answers may vary. Students may suggest that she use a number dodecahedron with 12 sides. Some students may suggest that she flip a coin, roll the number decahedron and use a spinner with 3 or 4 sections. Some students may suggest that she use a deck of cards and assign each student a card from the deck.

Use the following information to answer questions 8-13.

In one particular class, Mrs. Alexander assigned combinations with Heads and a prime number to only boys and combinations with Tails to only girls.

10. What is the sample space for this class?

Head, 1	*Head, 6*	*Tail, 1*	*Tail, 6*
Head, 2	*Head, 7*	*Tail, 2*	*Tail, 7*
Head, 3	*Head, 8*	*Tail, 3*	*Tail, 8*
Head, 4	*Head, 9*	*Tail, 4*	*Tail, 9*
Head, 5	*Head, 10*	*Tail, 5*	*Tail, 10*

(continue: The Helper Dilemma)

11. What can you conclude about this particular class? Explain.

Not all of the combinations in the sample space will be used for this class. This class has more girls than boys since more combinations are assigned to girls than boys.

12. Which gender is most likely to be the helper? Explain.

A girl is most likely to be the helper since more combinations are assigned to girls than boys.

13. What is the probability of a girl being the helper? Explain.

There is a $\frac{1}{2}$ *chance of getting a tail and a* $\frac{10}{10}$ *chance of getting a number on the decahedron. Combine the probabilities using multiplication,* $\frac{1}{2} \cdot \frac{10}{10}$, *to get a* $\frac{10}{20} = \frac{1}{2}$ *chance of getting a girl helper.*

14. What is the probability of a boy being the helper? Explain.

There is a $\frac{1}{2}$ *chance of getting a head and a* $\frac{4}{10}$ *chance of getting a prime number on the number decahedron. Combine the probabilities using multiplication,* $\frac{1}{2} \cdot \frac{4}{10}$, *to get a* $\frac{4}{20} = \frac{1}{5}$ *chance of getting a boy helper.*

15. How could Mrs. Alexander change or modify her procedure for finding a helper in this class to eliminate the extra combinations? Explain.

Answers may vary. Mrs. Alexander could use the coin and a bag of marbles with 4 different colors for the boys or a spinner with 4 equal sections.

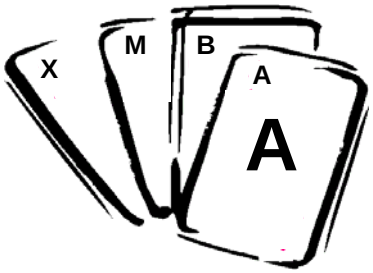
The Choir Helper – (Possible Answers)

The choir teacher, Mr. Roberts, heard Mrs. Alexander in the teacher's lounge describe her method for assigning a helper. He thought the idea would be a big help in his classes. Since his choir classes sometimes have between 45 and 50 students and no students can be assigned the same "code", Mr. Roberts can not use the coin and 10-sided number decahedron. Mrs. Alexander gave Mr. Roberts 8 different items that he could use to assign helpers in his class.

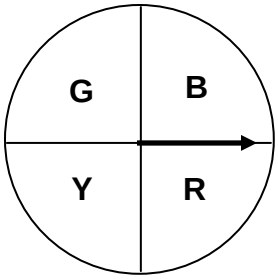
A Coin

A Six-Sided Number Cube

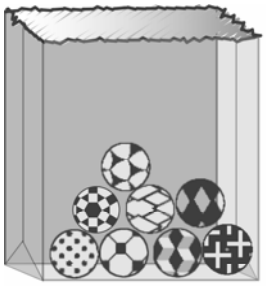
A 10-sided Number Decahedron with the numbers 1-10



A Set of Alphabet Cards A-Z

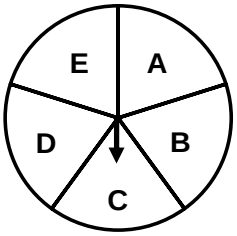


A Spinner



A Bag of 8 Different Marbles

A 12-sided Number Dodecahedron with the numbers 1-12



A Spinner

(continue: The Choir Helper)

1. Help Mr. Roberts pair the items together that he can use them to assign helpers. There will be 4 pairs. Justify your reasoning for each pair made and tell how many assignments for helpers could be made from each pair.

   Pair 1: A bag of 8 marbles and the 6-sided number cube (48 assignments)

   Pair 2: The coin and Set of Alphabet Cards (52 assignments)

   Pair 3: The spinner of colors and the 12-sided number dodecahedron (48 assignments)

   Pair 4: The spinner with letters and the 10-sided number decahedron (50 assignments)

2. Choose one of the pairs of items above and simulate the event for 50 trials. Create a frequency table to record your results.

   Answers may vary experiment to experiment.

3. Create a table in **The Teacher Helper** document under the tab titled **The Choir Helper** to organize the results.

4. Use the spreadsheet to predict the results if the event had been simulated for 100 trials? 250 trials? Make a separate column for each and use formulas to make predictions.

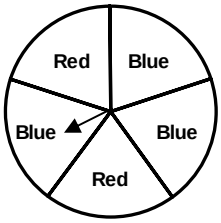
   Answers may vary, but formulas should include that the results in #3 are multiplied by a scale factor of 2 for 100 trials and a scale factor of 5 for 250 trials.

5. Print the document.

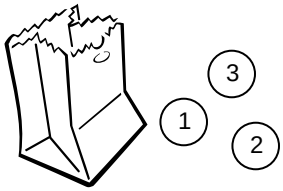
Simulation – (Possible Answers)

Use the following items to simulate an experiment.

A Coin



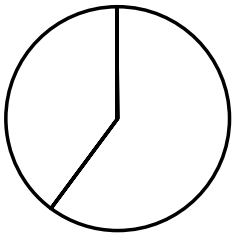
A Spinner



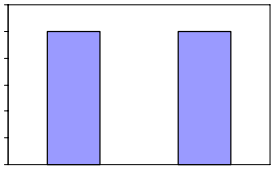
A Bag of 3 Marbles
Numbered 1-3

Which of the following graphs best represents the results of the experiment? Justify your reasoning.

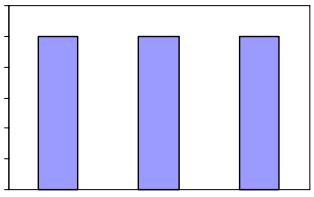
A.



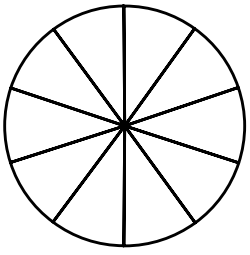
C.



B.



D.

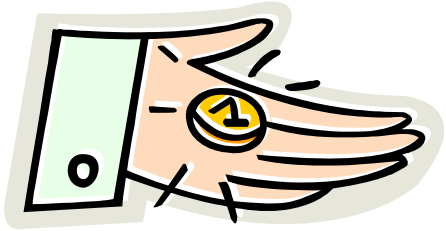


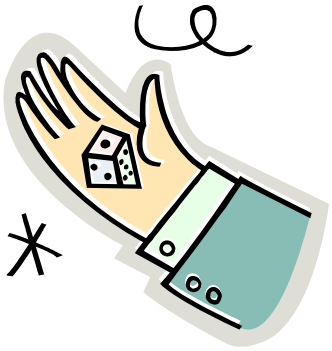
Answer: The graph in A best represents the experiment. In the experiment, the spinner has more blue than red. A circle graph representing the results of blue to red would show a larger section for blue.

Teacher Helper

Mrs. Alexander assigns the job of Teacher Helper in her class by flipping a coin and rolling a 10-sided number decahedron. Each student in her class is assigned a combination of a head or tail and a number from the decahedron. Students in the same class do not share the same combination.

- If all the possible combinations are assigned, how many students are in Mrs. Alexander's class?
- What are the possible combinations?





The Helper Dilemma

1. Use a coin and a 10-sided number decahedron to simulate the experiment 40 times. Record your results in the frequency table.

Combination	Tally	Frequency
Head, 1		
Head, 2		
Head, 3		
Head, 4		
Head, 5		
Head, 6		
Head, 7		
Head, 8		
Head, 9		
Head, 10		

Combination	Tally	Frequency
Tail, 1		
Tail, 2		
Tail, 3		
Tail, 4		
Tail, 5		
Tail, 6		
Tail, 7		
Tail, 8		
Tail, 9		
Tail, 10		

2. Transfer your information into the **The Teacher Helper** document. Follow the instructions in the orange boxes numbered 1-6.

3. Create a graph to represent the Theoretical Probability in Column B.
   - Highlight the Combinations (i.e. Head, 1) in Column A along with the data in the green cells in Column B.
   - Go to Insert Chart.
     - Choose "doughnut" for the chart type on the left-hand side.
     - Click on next twice and type in the title "Theoretical Probability."
     - Click on the tab that reads "Legend." Click in the box next to "Show Legend" so that the check mark disappears.
     - Click on the tab that reads "Data Labels." Click inside the boxes next to "Category Name and Value" so that a check mark appears in both boxes.
     - Click on finish.
     - Click and hold inside the chart. Drag the chart below the first set of data.
     - Enlarge the chart by clicking on a corner and dragging to the desired size.

(continue: The Helper Dilemma)

4. Create a graph to represent the Experimental Probability in Column I (include the Combinations such as Head, 1). Follow the same instructions as #3 except highlight the information in Columns H and I and use the title "Experimental Probability." Drag the chart next to the Theoretical Probability Chart, the first chart.

5. Print the document. Be sure to preview the pages to be printed. You may need to adjust margins so that you only print 1 or 2 pages.

6. How were the experimental and theoretical probabilities the same? Explain.

7. How were the experimental and theoretical probabilities different? Explain.

8. If the fractions were changed to percents, what would you expect the percents to total and why?

9. If Mrs. Alexander has more students enrolled in her class, how can she change or modify her procedure for finding a helper?

Use the following information to answer questions 8-13.

In one particular class, Mrs. Alexander assigned combinations with Heads and a prime number to only boys and combinations with Tails to only girls.

10. What is the sample space for this class?

11. What can you conclude about this particular class? Explain.

(continue: The Helper Dilemma)

12. Which gender is most likely to be the helper? Explain.

13. What is the probability of a girl being the helper? Explain.

14. What is the probability of a boy being the helper? Explain.

15. How could Mrs. Alexander change or modify her procedure for finding a helper in this class to eliminate the extra combinations? Explain.

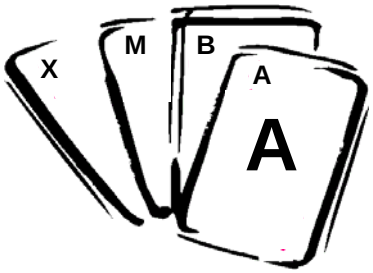
The Choir Helper

The choir teacher, Mr. Roberts, heard Mrs. Alexander in the teacher's lounge describe her method for assigning a helper. He thought the idea would be a big help in his classes. Since his choir classes sometimes have between 45 and 50 students and no students can be assigned the same "code," Mr. Roberts cannot use the coin and 10-sided number decahedron. Mrs. Alexander gave Mr. Roberts 8 different items that he could use to assign helpers in his class.

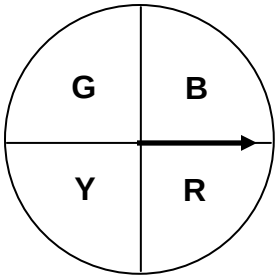
A Coin

A Six-Sided Number Cube

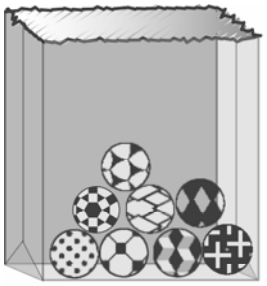
A 10-sided Number Decahedron with the numbers 1-10



A Set of Alphabet Cards A-Z

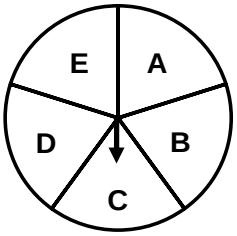


A Spinner



A Bag of 8 Different Marbles

A 12-sided Number Dodecahedron with the numbers 1-12



A Spinner

(continue: The Choir Helper)

1. Help Mr. Roberts pair the items together that he can use to assign helpers. There will be 4 pairs. Justify your reasoning for each pair made and tell how many assignments for helpers could be made from each pair.

2. Choose one of the pairs of items above and simulate the event for 50 trials. Create a frequency table to record your results.

3. Create a table in **The Teacher Helper** document under the tab titled **The Choir Helper** to organize the results.

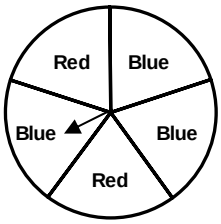
4. Use the spreadsheet to predict the results if the event had been simulated for 100 trials? 250 trials? Make a separate column for each and use formulas to make predictions.

5. Print the document.

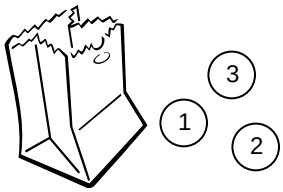
Simulation

Use the following items to simulate an experiment.

A Coin



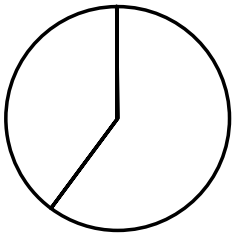
A Spinner



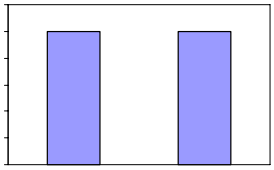
A Bag of 3 Marbles
Numbered 1-3

Which of the following graphs best represents the results of the experiment? Justify your reasoning.

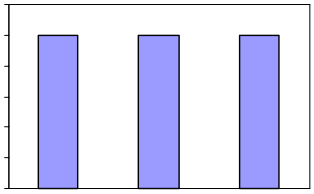
A.



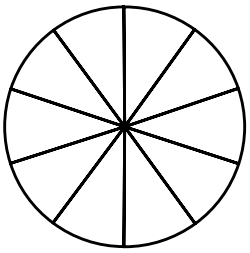
C.



B.



D.



1. Corbyn has a standard code of dress at his school. He can wear a white or green shirt with navy or khaki pants. He had 3 white shirts and 2 green shirts in his shirt drawer and 1 pair of navy pants and 3 pairs khaki pants in his pants drawer. What is the probability that Corbyn will reach in both drawers, without looking, and get a white shirt and navy pants?

   A $\frac{17}{20}$

   B $\frac{4}{9}$

   C $\frac{3}{25}$

   D $\frac{3}{20}$

2. A 6-sided number cube, a spinner divided into 3 equal parts labeled A, A, B, and a coin are being used for an experiment. Ozzie calculated the theoretical probability of an event where the number cube was rolled, coin tossed, and spinner spun. His calculation was $\frac{1}{3} \cdot \frac{1}{2} \cdot \frac{2}{3} = \frac{2}{18} = \frac{1}{9}$. For which of the following events did Ozzie calculate the probability?

   A P(even number, head, B)

   B P(1 or 2, head, A)

   C P(prime number, tail, A)

   D P(odd number, tail, A)

3. The letters of the word WINNER are cut apart and placed in a bag. A letter was drawn from the bag and a coin tossed at the same time. Results were recorded and the letter was placed back into the bag. Which of the following could NOT be used to represent the experimental data?

   A Venn diagram

   B Bar graph

   C Circle graph

   D Line Plot

4. A container of markers containing 3 red, 1 yellow, 2 green and 4 blue are placed at the map center in social studies. The rule is you can only use one marker at a time so that everyone will have a marker to use. What is the probability of reaching into the container without looking for each use and getting a red marker, a blue marker and then a yellow marker?

   A $\frac{3}{250}$

   B $\frac{12}{30}$

   C $\frac{12}{100}$

   D $\frac{8}{10}$

Mathematics

7.10 The student recognizes that a physical or mathematical model can be used to describe the experimental and theoretical probability of real –life. The student is expected to

(B) Find the probability of independent events.

7.11 The Students understands that the way a set of data is displayed influences its interpretation. The student is expected to

(A) to select and use an appropriate representation for presenting and displaying relationships among collected data, including line plot, line graph, bar graph, stem and leaf plot, circle graph, and Venn diagrams, and justify the selection.

(B) make inferences and convincing arguments based on an analysis of given or collected data.

Technology Applications

The student demonstrates knowledge and appropriate use of hardware components, software programs, and their connections. The student is expected to

(1)(B) compare, contrast, and appropriately use the various input, processing, output, and primary/secondary storage devices.

(1)(C) demonstrate the ability to select and use software for a defined task according to quality, appropriateness, effectiveness, and efficiency.

The student uses appropriate computer-based productivity tools to create and modify solutions to problems. The student is expected to

(7)(H) use interactive virtual environments, appropriate to level, such as virtual reality or simulations.

Materials

Advances Preparation:

- Make the **Teacher Helper** transparency.

For each student:

- **The Helper Dilemma** activity sheet
- **The Choir Helper** activity sheet
- **Simulation** activity sheet
- TI-73 calculator

For each student group of students:

- Coin
- 10-sided number decahedron

For whole class demonstration:

- Transparency of **Teacher Helper**

ENGAGE

The Engage portion of the lesson is designed to create student interest in the concepts addressed. Technology is not used in this phase since the focus of this activity is to remind students of sample spaces and probability. This part of the lesson is designed for whole group instruction and groups of 2 students.

1. Display the **Teacher Helper** transparency on the overhead.
2. Read the problem as a class and ask students to take a moment to think about the problem on their own.
3. Prompt students to work with a partner to compile their thoughts and answer the questions.
4. Debrief using the facilitation questions.
5. Extend the discussion to find other probabilities such as the probability of getting an even number, the number 11, etc.

Facilitation Questions – Engage Phase

- How many students are in Mrs. Alexander's class?
  20
- How do you know?
  Answers may vary. This will hopefully lead into a discussion on sample spaces.
- What is a sample space?
  A sample space is the set of all possible outcomes for a given scenario.
- What is the sample space for this scenario?

Heads, 1	*Heads, 6*	*Tails, 1*	*Tails, 6*
Heads, 2	*Heads, 7*	*Tails, 2*	*Tails, 7*
Heads, 3	*Heads, 8*	*Tails, 3*	*Tails, 8*
Heads, 4	*Heads, 9*	*Tails, 4*	*Tails, 9*
Heads, 5	*Heads, 10*	*Tails, 5*	*Tails, 10*

- Are all of the possibilities equally likely? Why?
  Yes, there is only one head and one tail. Also, each number occurs only one time.
- What is the probability of the student assigned to Head, 6 being the helper?
  Have students refer back to the sample space. There is a $\frac{1}{20}$ *chance for the student with a Head, 6 to be the helper. Connect the sample space to finding the independent probability.*

EXPLORE

The Explore portion of the lesson provides the student with an opportunity to be actively involved in the exploration of the mathematical concepts addressed. This part of the lesson is designed for groups of 2 students.

1. Distribute a TI-73 to each student.
2. Distribute **The Helper Dilemma** activity sheet to each student.
3. The students should perform the experiment and record results.
4. Use the facilitation questions when students need help.

Facilitation Questions – Explore Phase

- How does the calculator help you generate the data?
  Answers may vary. The probability simulator performs the trials for you.
- How is using the calculator more beneficial than actually flipping the coin and rolling the number decahedron?
  Answers may vary. The calculator may be more reliable since it takes out the human error factor.
- How is using the calculator less beneficial than using the objects to simulate the experiment?
  Answers may vary. Lead students in a discussion that batches of calculators are programmed to start at the same random generating point. The data collected may be less random than data simulated with the actual objects.
- What do you know about the problem?
  Answers may vary. Have students verbalize the parts of the problem they know.
- What do you need to know?
  Answers may vary. Have students verbalize the parts of the problem they need to know through questioning.
- Have you worked problems like this before?
  Answers may vary. Relate the problem to prior learning and prior experiences.

EXPLAIN

The teacher directs the Explain portion of the lesson to allow the students to formalize their understanding of the TEKS addressed in the lesson.

1. Debrief **The Helper Dilemma**.
2. Discuss notation for probability of compound events (i.e. P(head, one)).

Facilitation Questions – Explain Phase

- What is the difference between theoretical and experimental probability?
  Answers may vary. Take this opportunity to review these topics.
- How were the experimental and theoretical probabilities the same?
  Answers may vary. Depending on the experiment, some may say that the experimental probabilities were close to being equally distributed.
- How were the experimental and theoretical probabilities different?
  Answers may vary. Depending on the experiment, some of the combinations may have occurred more than others. Possibly discuss at this time how Mrs. Alexander should keep track of who is helper so that when repeats occur, she knows to flip the coin and roll the number decahedron again.
- If the fractions were changed to percents, what would you expect the percents to total and why?
  Answers may vary. Lead students to the understanding that the experiment is a whole event, so that the percents would add to 100% and the fractions to 1 whole.
- If Mrs. Alexander has more students enrolled in her class, how can she change or modify her procedure for finding a helper?
  Answers may vary. Students may suggest that she use a number polyhedron with 12 sides. Some students may suggest that she flip a coin, roll the polyhedron and use a spinner with 3 or 4 sections. Some students may suggest that she use a deck of cards and assign each student a card from the deck.
- What can you conclude about the class where Mrs. Alexander assigned tails to girls and heads and prime numbers to boys?
  Answers may vary. Not all of the combinations in the sample space will be used for this class. This class has more girls than boys since more combinations are assigned to girls than boys.
- How could Mrs. Alexander change or modify her procedure for finding a helper in this class to eliminate the extra combinations?
  Answers may vary. Mrs. Alexander could use the coin and a bag of marbles with 4 different colors for the boys or a spinner with 4 equal sections.
- In a previous question you were asked, "If Mrs. Alexander has more students enrolled in her class, how can she change or modify her procedure for finding a helper?" If 3 items were used to find the helper, for example, a coin, a number polyhedron and a spinner, how would the results be affected?
  Answers may vary. Discuss that a third item would increase outcomes.
- Do the items always have to yield equally likely results?
  Answers may vary. Discuss with students that they do not. Have students give examples where the outcomes are not equally likely.
- Are there times when the technology made the task easier? Why?
  Answers may vary. Some students may say that simulating the events using the calculator made the task easier.

Facilitation Questions – Explain Phase

- Are there times when the technology made the task more difficult? Why?
  Answers may vary. Some students may say recording the data into the lists and creating the graph would have been easier using only paper and pencil.

ELABORATE

The Elaborate portion of the lesson provides an opportunity for the student to apply the concepts of the TEKS to a new situation. This part of the lesson is designed for individual investigation.

1. Distribute a TI-73 calculator and **The Choir Helper** activity sheet to each student.
2. Have students should work individually to solve the problems using the calculator to help them.
3. Ask Facilitation Questions as needed.
4. Debrief by going over the worksheet.

Facilitation Questions – Elaborate Phase

- How do you use the calculator to help you?
  Possible answer: The calculator can be used to simulate the trials.
- How is this activity similar to **The Helper Dilemma**?
  Possible answer: Both activities used two items for simulation.
- How is this activity different?
  Possible answer: The sample spaces were different.
- How do you make predictions from results?
  Include a discussion here of scale factors. The simulation was for 50 trials, so to predict results for 100 trials use a scale factor of 2 (multiply by 2).
- How could you use a graph to show that the results for 50 trials, 100 trials, 250 trials, etc. are proportional?
  Answers may vary. Discuss with students that a graph could be made charting each individual outcome. For example, chart the results for P(tail, letter A) for 50 trials by letting the List 1 represent trials and the List 2 represent outcomes. Other outcomes for the same probability could be entered into the lists and then graphed. A discussion of the data points should follow. The points should appear to be in a straight line that would travel through the origin. Thus, the data is proportional.

EVALUATE

The Evaluate portion of the lesson provides the student with an opportunity to demonstrate his or her understanding of the TEKS addressed in the lesson.

1. Distribute **Simulation** activity sheet to each student.
2. Upon completion of **Simulation** activity sheet, the teacher should use a rubric to assess student understanding of the concepts addresses in this lesson.

Answers and Error Analysis for selected response questions:

Question Number	TEKS	Correct Answer	Conceptual Error	Conceptual Error	**Procedural** Error	Procedural Error	**Guess**
1	7.10B	D	A		C		B
2	7.11B	B	A	D			C
3	7.11A	A	B	C			D
4	7.10B	A	D		B	C	

The Helper Dilemma – (Possible Answers)

1. Use two TI-73 calculators to simulate the probability. Each student should hold a calculator. One calculator should be used to simulate the coin toss and the other used to simulate rolling a 10-sided number decahedron. Follow the instructions below to simulate the events with the calculators. Combine the results and place a tally mark in the frequency table below. Perform the experiment 40 times.

Coin Toss
APPS 7: Prob Sim Press any key 2. Toss Coin Toss (Window)

Roll Dice
APPS 7: Prob Sim Press any key 1. Roll Dice Set (Zoom) Sides: 10 OK (Graph) Roll (Window)

Combination	Tally	Frequency
Head, 1		
Head, 2		
Head, 3		
Head, 4		
Head, 5		
Head, 6		
Head, 7		
Head, 8		
Head, 9		
Head, 10		

Combination	Tally	Frequency
Tail, 1		
Tail, 2		
Tail, 3		
Tail, 4		
Tail, 5		
Tail, 6		
Tail, 7		
Tail, 8		
Tail, 9		
Tail, 10		

(continue: The Helper Dilemma)

2. Graph the data using the instructions below. Sketch the resulting graph.

Graph Data	Sketch graph here.
Enter the frequency data in L2 of the LIST feature. 2nd Plot (Y=) 1: Plot 1 On Type: Pie Chart Graph	*Graph will vary, but should match results in table.*

3. Find the experimental probability for each.

Answers may vary experiment by experiment.

Combination	Experimental Probability
Head, 1	
Head, 2	
Head, 3	
Head, 4	
Head, 5	
Head, 6	
Head, 7	
Head, 8	
Head, 9	
Head, 10	

Combination	Experimental Probability
Tail, 1	
Tail, 2	
Tail, 3	
Tail, 4	
Tail, 5	
Tail, 6	
Tail, 7	
Tail, 8	
Tail, 9	
Tail, 10	

(continue: The Helper Dilemma)

4. How were the experimental and theoretical probabilities the same? Explain.
   Answers may vary. Depending on the experiment, some may say that the experimental probabilities were close to being equally distributed.

5. How were the experimental and theoretical probabilities different? Explain.
   Answers may vary. Depending on the experiment, some of the combinations may have occurred more than others. Possibly discuss at this time how Mrs. Alexander should keep track of who is helper so that when repeats occur, she knows to flip the coin and roll the decahedron again.

6. If the fractions were changed to percents, what would you expect the percents to total and why?
   Answers may vary. Lead students to the understanding that the experiment is a whole event, so that the percents would add to 100% and the fractions to 1 whole.

7. If Mrs. Alexander has more students enrolled in her class, how can she change or modify her procedure for finding a helper?
   Answers may vary. Students may suggest that she use a number polyhedron with 12 sides. Some students may suggest that she flip a coin, roll the polyhedron and use a spinner with 3 or 4 sections. Some students may suggest that she use a deck of cards and assign each student a card from the deck.

Use the following information to answer questions 8-13.

In one particular class, Mrs. Alexander assigned combinations with Heads and a prime number to only boys and combinations with Tails to only girls.

8. What is the sample space for this class?

Head, 1	*Head, 6*	*Tail, 1*	*Tail, 6*
Head, 2	*Head, 7*	*Tail, 2*	*Tail, 7*
Head, 3	*Head, 8*	*Tail, 3*	*Tail, 8*
Head, 4	*Head, 9*	*Tail, 4*	*Tail, 9*
Head, 5	*Head, 10*	*Tail, 5*	*Tail, 10*

9. What can you conclude about this particular class? Explain.
   Not all of the combinations in the sample space will be used for this class. This class has more girls than boys since more combinations are assigned to girls than boys.

10. Which gender is most likely to be the helper? Explain.
    A girl is most likely to be the helper since more combinations are assigned to girls than boys.

(continue: The Helper Dilemma)

11. What is the probability of a girl being the helper? Explain.

There is a $\frac{1}{2}$ chance of getting a tail and a $\frac{10}{10}$ chance of getting a number on the decahedron. Combine the probabilities using multiplication, $\frac{1}{2} \cdot \frac{10}{10}$, to get a $\frac{10}{20} = \frac{1}{2}$ chance of getting a girl helper.

12. What is the probability of a boy being the helper? Explain.

There is a $\frac{1}{2}$ chance of getting a head and a $\frac{4}{10}$ chance of getting a prime number on the decahedron. Combine the probabilities using multiplication, $\frac{1}{2} \cdot \frac{4}{10}$, to get a $\frac{4}{20} = \frac{1}{5}$ chance of getting a boy helper.

13. How could Mrs. Alexander change or modify her procedure for finding a helper in this class to eliminate the extra combinations? Explain.

Answers may vary. Mrs. Alexander could use the coin and a bag of marbles with 4 different colors for the boys or a spinner with 4 equal sections.

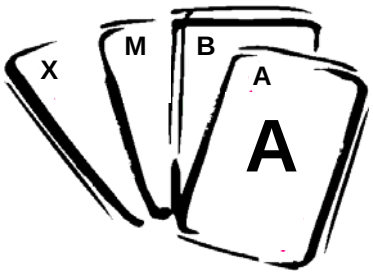
The Choir Helper – (Possible Answers)

The choir teacher, Mr. Roberts, heard Mrs. Alexander in the teacher's lounge describe her method for assigning a helper. He thought the idea would be a big help in his classes. Since his choir classes sometimes have between 45 and 50 students and no students can be assigned the same "code," Mr. Roberts cannot use the coin and 10-sided number decahedron. Mrs. Alexander gave Mr. Roberts 8 different items that he could use to assign helpers in his class.

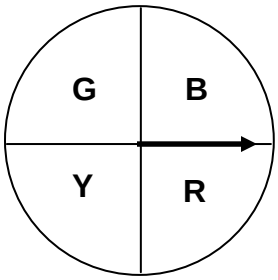
A Coin

A Six-Sided Number Cube

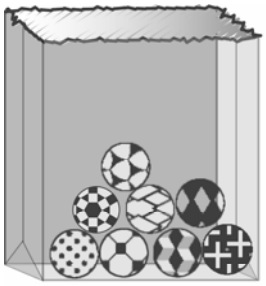
A 10-sided Number Decahedron with the numbers 1-10



A Set of Alphabet Cards A-Z

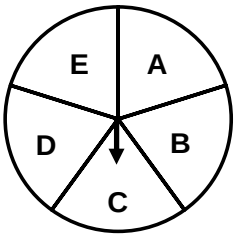


A Spinner



A Bag of 8 Different Marbles

A 12-sided Number Dodecahedron with the numbers 1-12



A Spinner

(continue: The Choir Helper)

1. Help Mr. Roberts pair the items together that he can use to assign helpers. There will be 4 pairs. Justify your reasoning for each pair made and tell how many assignments for helpers could be made from each pair.

   Pair 1: A bag of 8 marbles and the 6-sided number cube (48 assignments)

   Pair 2: The coin and set of Alphabet Cards (52 assignments)

   Pair 3: The spinner of colors and the 12-sided number dodecahedron (48 assignments)

   Pair 4: The spinner with letters and the 10-sided number decahedron (50 assignments)

2. Choose one of the pairs of items above and describe how to simulate the event using the calculator.

   Answers may vary experiment to experiment.

3. Use the plan outlined in #2 to simulate the event for 50 trials. Create a table to record the results.

   Answers may vary experiment to experiment.

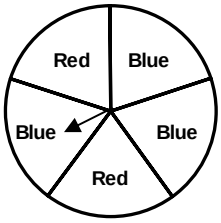
4. From the above results, predict the results if the event had been simulated for 100 trials.

   Answers may vary, but should include that the results in #3 should be multiplied by a scale factor of 2.

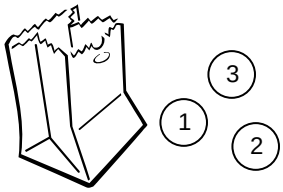
Simulation – (Possible Answers)

Use the following items to simulate an experiment.

A Coin



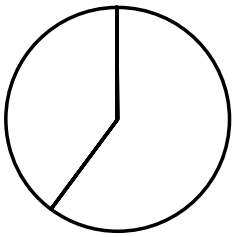
A Spinner



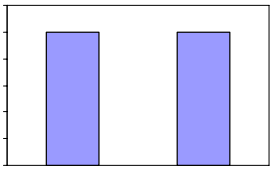
A Bag of 3 Marbles
Numbered 1-3

Which of the following graphs best represents the results of the experiment? Justify your reasoning.

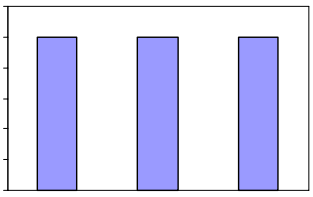
A.



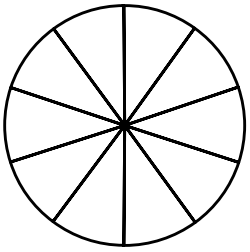
C.



B.



D.

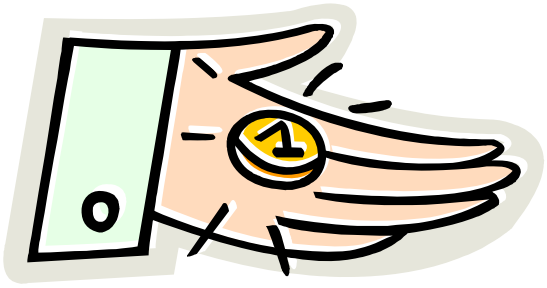


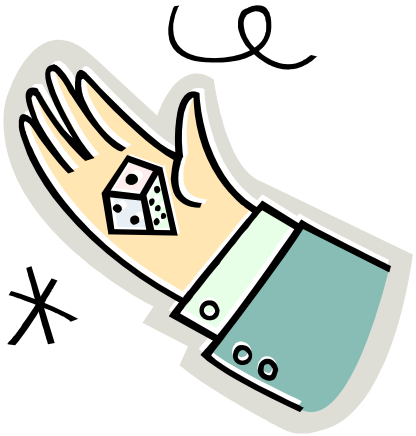
Answer: The graph in A best represents the experiment. In the experiment, the spinner has more blue than red. A circle graph representing the results of blue to red would show a larger section for blue.

Teacher Helper

Mrs. Alexander assigns the helping job in her class by flipping a coin and rolling a 10-sided number decahedron. Each student in her class is assigned a combination of a head or tail and a number from the decahedron. Students in the same class do not share the same combination.

- If all the possible combinations are assigned, how many students are in Mrs. Alexander's class?
- What are the possible combinations?





The Helper Dilemma

1. Use two TI-73 calculators to simulate the probability. Each student should hold a calculator. One calculator should be used to simulate the coin toss and the other used to simulate rolling a 10-sided number decahedron. Follow the instructions below to simulate the events with the calculators. Combine the results and place a tally mark in the frequency table below. Perform the experiment 40 times.

Coin Toss
APPS 7: Prob Sim Press any key 4. Toss Coin Toss (Window)

Roll Dice
APPS 7: Prob Sim Press any key 3. Roll Dice Set (Zoom) Sides: 10 OK (Graph) Roll (Window)

Combination	Tally	Frequency
Head, 1		
Head, 2		
Head, 3		
Head, 4		
Head, 5		
Head, 6		
Head, 7		
Head, 8		
Head, 9		
Head, 10		

Combination	Tally	Frequency
Tail, 1		
Tail, 2		
Tail, 3		
Tail, 4		
Tail, 5		
Tail, 6		
Tail, 7		
Tail, 8		
Tail, 9		
Tail, 10		

2. Graph the data using the instructions below. Sketch the resulting graph.

Graph Data

Enter the frequency data in L2 of the LIST feature.

2nd
Plot (Y=)
1: Plot 1
On
Type: Pie Chart
Graph

Sketch graph here.

3. Find the experimental probability for each.

Combination	Experimental Probability
Head, 1	
Head, 2	
Head, 3	
Head, 4	
Head, 5	
Head, 6	
Head, 7	
Head, 8	
Head, 9	
Head, 10	

Combination	Experimental Probability
Tail, 1	
Tail, 2	
Tail, 3	
Tail, 4	
Tail, 5	
Tail, 6	
Tail, 7	
Tail, 8	
Tail, 9	
Tail, 10	

(continue: The Helper Dilemma)

4. How were the experimental and theoretical probabilities the same? Explain.

5. How were the experimental and theoretical probabilities different? Explain.

6. If the fractions were changed to percents, what would you expect the percents to total and why?

7. If Mrs. Alexander has more students enrolled in her class, how can she change or modify her procedure for finding a helper?

Use the following information to answer questions 8-13.

In one particular class, Mrs. Alexander assigned combinations with Heads and a prime number to only boys and combinations with Tails to only girls.

8. What is the sample space for this class?

9. What can you conclude about this particular class? Explain.

10. Which gender is most likely to be the helper? Explain.

11. What is the probability of a girl being the helper? Explain.

(continue: The Helper Dilemma)

12. What is the probability of a boy being the helper? Explain.

13. How could Mrs. Alexander change or modify her procedure for finding a helper in this class to eliminate the extra combinations? Explain.

The Choir Helper

The choir teacher, Mr. Roberts, heard Mrs. Alexander in the teacher's lounge describe her method for assigning a helper. He thought the idea would be a big help in his classes. Since his choir classes sometimes have between 45 and 50 students and no students can be assigned the same "code," Mr. Roberts cannot use the coin and 10-sided number decahedron. Mrs. Alexander gave Mr. Roberts 8 different items that he could use to assign helpers in his class.

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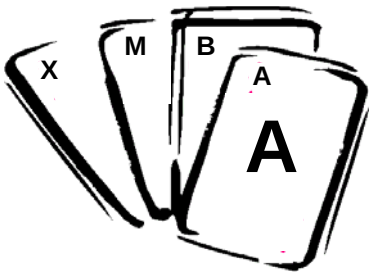
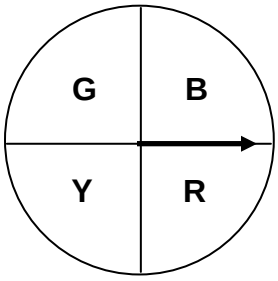
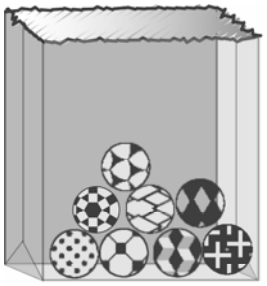


A Set of Alphabet Cards A-Z



A Spinner



A Bag of 8 Different Marbles

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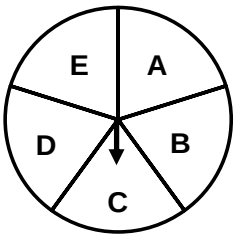


A Spinner

(continue: The Choir Helper)

1. Help Mr. Roberts pair the items together that he can use to assign helpers. There will be 4 pairs. Justify your reasoning for each pair made and tell how many assignments for helpers could be made from each pair.

2. Choose one of the pairs of items above and describe how to simulate the event using the calculator.

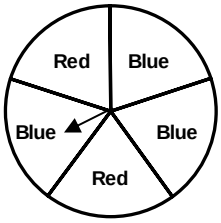
3. Use the plan outlined in #2 to simulate the event for 50 trials. Create a table to record the results.

4. From the above results, predict the results if the event had been simulated for 100 trials.

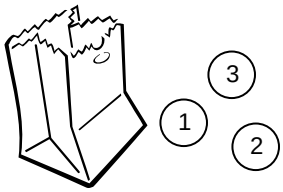
Simulation

The following items are being used to simulate an experiment.

A Coin



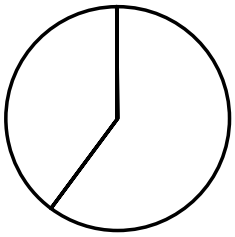
A Spinner



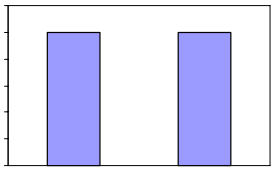
A Bag of 3 Marbles
Numbered 1-3

Which of the following graphs best represent the results of the experiment? Justify your reasoning

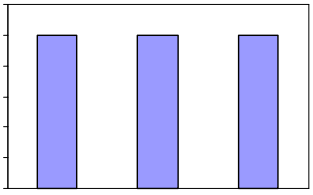
A.



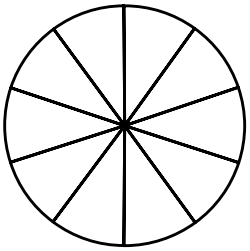
C.



B.



D.



1. Corbyn has a standard code of dress at his school. He can wear a white or green shirt with navy or khaki pants. He had 3 white shirts and 2 green shirts in his shirt drawer and 1 pair of navy pants and 3 pairs of khaki pants in his pants drawer. What is the probability that Corbyn will reach in both drawers, without looking, and get a white shirt and navy pants?

   A $\frac{17}{20}$

   B $\frac{4}{9}$

   C $\frac{3}{25}$

   D $\frac{3}{20}$

2. A 6-sided number cube, a spinner divided into 3 equal parts labeled A, A, B, and a coin are used for an experiment. Ozzie calculated the theoretical probability of an event where the number cube was rolled, coin tossed, and spinner spun. His calculation was $\frac{1}{3}\cdot\frac{1}{2}\cdot\frac{2}{3}=\frac{2}{18}=\frac{1}{9}$. For which of the following events did Ozzie calculate the probability?

   A P(even number, head, B)

   B P(1 or 2, head, A)

   C P(prime number, tail, A)

   D P(odd number, tail, A)

3. The letters of the word WINNER are cut apart and placed in a bag. A letter was drawn from the bag and a coin tossed at the same time. Results were recorded and the letter was placed back into the bag. Which of the following could NOT be used to represent the experimental data?

   A Venn diagram

   B Bar graph

   C Circle graph

   D Line Plot

4. A container of markers containing 3 red, 1 yellow, 2 green and 4 blue are placed at the map center in social studies. The rule is you can only use one marker at a time so that everyone will have a marker to use. What is the probability of reaching into the container without looking for each use and getting a red marker, a blue marker and then a yellow marker?

   A $\frac{3}{250}$

   B $\frac{12}{30}$

   C $\frac{12}{100}$

   D $\frac{8}{10}$